Earnings Inequality and the Minimum Wage: Evidence from Brazil^{*}

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Abstract

We assess the extent to which a rise in the minimum wage can account for three facts characterizing a large decline in earnings inequality in Brazil from 1996–2012: (i) the decline is bottom-driven yet wide-spread; (ii) lower-tail inequality is negatively correlated with the bindingness of the minimum wage across Brazilian states over time; and (iii) a large share of the decline is due to a compression in the returns to firm and worker characteristics in pay. To this end, we build a general equilibrium search model with heterogeneous firms engaging in monopsonistic competition for heterogeneous workers. The rise in the minimum wage in our model explains 70 percent of the observed decline in the variance of log earnings, while being consistent with the above three facts.

Keywords: Earnings Inequality, Worker and Firm Heterogeneity, Minimum Wage, Equilibrium Search Model

JEL classification: D33, E24, J08, J31, J38, N36

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1 Introduction

Earnings inequality has become central to recent debates in academic and policy circles.¹ A majority of respondents to an international survey identified government policies as the most frequently cited reason for prevailing inequality levels ([Pew Research Center, 2014](#)). This raises the question: How large are the effects of labor market and other policies on the distribution of income?

To shed light on this question, we study Brazil as an economy that experienced a rapid decline in earnings inequality between 1996 and 2012. Starting at high initial inequality levels, Brazil saw a 26 log points (or 35 percent) fall in the variance of log earnings over this period. By comparison, in the U.S. the same measure increased by six log points (or 12 percent) during those years.² Concurrent with Brazil’s remarkable inequality decline, the country’s minimum wage rose by 119 percent in real terms. Yet, given the ongoing debate in the literature about consequences of the minimum wage, the extent to which these two trends are related is far from clear.³

The main contribution of this paper is to quantify the effect of the rise in the minimum wage on Brazil’s inequality evolution. To this end, we use administrative matched employer-employee data to review and extend key empirical patterns that we document in detail in [Alvarez, Benguria, Engbom, and Moser \(2016\)](#). Specifically, the three facts characterizing Brazil’s earnings inequality decline that we address in the current paper are: (i) the decline is more pronounced towards the bottom of the distribution; (ii) one quarter of the decline stems from an increase in relative pay at less productive firms; and (iii) another quarter of the decline is attributable to falling pay differences due to worker heterogeneity, largely driven by decreasing returns to education and age. Hence, any candidate explanation for Brazil’s inequality evolution needs to generate compression in the earnings distribution driven from the bottom with changes in the returns to firm and worker characteristics playing a prominent role.⁴

To assess the extent to which the rise in the minimum wage can account for these facts, we build a model of frictional wage dispersion based on the canonical search framework by [Burdett and Mortensen \(1998\)](#). Motivated by our empirical findings, we extend this framework in a

¹ Examples of recent research concerned with earnings inequality include [Atkinson and Bourguignon, eds \(2015\)](#) for an overview of academic work, [OECD \(2015\)](#) for policy issues in a number of developed countries, and [IMF \(2015\)](#); [World Bank \(2013\)](#) for policy relating to emerging markets and developing economies.

² Inequality measures are defined over labor income for workers of age 18–64 using the *March Current Population Survey (CPS)* for the U.S.; and the *Relação Anual de Informações Sociais (RAIS)* for Brazil. See Appendix A.1 for details.

³ See [Flinn \(2010\)](#) for a selective survey of the literature.

⁴ In contrast, we show that the underlying distributions of firm and worker characteristics, notably firm productivity and workers’ educational attainment, became more dispersed over the period.

tractable way to include heterogeneity in both worker ability and firm productivity. The key feature of this environment is that the minimum wage indirectly affects higher parts of the earnings distribution. Because firms compete for workers on wages, higher productivity firms increase their equilibrium wage offers above the new minimum wage in order to poach and retain workers. Therefore, while the minimum wage has a direct impact on the least productive workers and firms in the economy, its effects will slowly fade out towards the top of the earnings distribution. These spillover effects open the door to the minimum wage qualitatively accounting for the three facts on Brazil's inequality decline.

We find that the minimum wage is also quantitatively successful at explaining the documented facts on the inequality evolution. To this end, we estimate key model parameters guiding labor mobility as well as heterogeneity in worker ability and firm productivity using indirect inference on the Brazilian microdata from 1996–2000. We then use the estimated model to simulate the effects of the observed minimum wage increase. The main result of this experiment is that 70 percent of the observed decline in the variance of log earnings are accounted for by the rise in the minimum wage. More than half of this decline is due to indirect effects of the minimum wage. In line with our empirical findings, the model generates significant compression up to the top decile of the earnings distribution. A sizable share of the overall inequality decline is due to a weaker productivity-pay gradient across firms, with the model generating a drop of 4.3 log points in the variance of log earnings due to this channel, relative to 5.0 log points in the data. Furthermore, the model predicts a fall in the dispersion of worker pay heterogeneity explaining an additional 6.2 log points fall in the variance of log earnings, compared to 5.6 log points in the data.⁵ Together, these results suggest that the minimum wage was an important driver behind Brazil's inequality decline.

A central feature of the model is the presence of spillover effects of the minimum wage on higher earnings ranks. Their source is the upwards-sloping labor supply curve faced by monopolistic firms under search frictions, creating a trade-off in wage setting between firm size and profitability. In equilibrium, more productive firms offer higher wages and workers gradually climb up a job ladder by moving to better-paying employers. Since the rates of poaching and retaining workers depend on a firm's rank in the wage offer distribution within each labor market

⁵ In the model, all of the decline in worker-specific pay is due to convergence in the returns to worker types. Also in the data, we find that all of the decline in worker heterogeneity explained by observable worker characteristics (age, education, and occupation) is due to decreasing returns to these characteristics.

segment, the minimum wage indirectly affects equilibrium wage posting strategies of all firms in the market. As the competitive pressure in response to a rise in the minimum wage is weaker for firms further up in the productivity distribution, the resulting productivity-pay gradient across firms is lower and earnings are less dispersed. Analogously, since lower ability worker are more likely and more intensely affected, the minimum wage also leads to compression in the relative rents captured by different worker types. We provide empirical evidence for the mechanism by identifying a job ladder across firms in the Brazilian microdata and show that, in line with the model predictions, this job ladder has become flatter as the minimum wage increased over time.

Related literature. Our work relates to three strands of the literature within the broad realm of understanding inequality in labor markets. The first strand is concerned with decomposing the determinants of earnings inequality into components relating to workers, firms, and other factors, and using this decomposition to understand changes in the earnings distribution over time. The seminal work in this area is that of [Abowd, Kramarz, and Margolis \(1999\)](#), henceforth AKM) who propose a two-way fixed effects framework controlling for unobserved worker and firm heterogeneity. They find that firms explain a significant share of earnings inequality levels in French linked employer-employee data (but do not study changes over time).⁶ In later work, [Card et al. \(2013\)](#) apply the same methodology to Germany and argue that firms explain a quarter to a third of the overall rise in earnings inequality in Germany. [Card et al. \(2015\)](#) use a static AKM framework to investigate the degree of differential sorting and rent sharing between male and female workers in Portugal. [Alvarez, Benguria, Engbom, and Moser \(2016\)](#) applies this methodology to understand Brazil's decline in inequality between 1988 and 2012, and find that falling inequality between firms in pay is an important component of this decline. Although not within an AKM framework, [Barth et al. \(2014\)](#) and [Song et al. \(2015\)](#) argue that changes in pay across firms were important in understanding the increase in wage dispersion in the U.S. during the last decades.

Second, our theoretical framework is closely related to the literature using search models to study wage dispersion. While work in this area goes back to at least [McCall \(1970\)](#), a more recent class of search models pioneered by [Burdett \(1978\)](#) and further developed by [Burdett and Mortensen \(1998\)](#) lays the foundation for our analysis of the effects of the minimum wage in a job

⁶In a later paper, [Abowd et al. \(2002\)](#) note that the approximate estimates obtained in AKM, particularly firm effects, were contaminated due to the approximation method used at the time. However, the authors note that their original conclusion, namely that person effects account for most wage variation in France, remains unchanged.

ladder environment. A rich body of follow-up work has used versions of this model to jointly study wage dispersion and labor dynamics ([van den Berg and Ridder, 1998](#); [Bontemps et al., 1999, 2000](#); [Mortensen, 2000, 2003](#); [Postel-Vinay and Robin, 2002](#); [Cahuc et al., 2006](#); [Jolivet et al., 2006](#); [de Araujo, 2014](#)). To this literature we contribute a tractable model of the minimum wage with heterogeneity in both firm productivities and worker abilities, an environment that previous research highlighted as important but difficult to study. In related work, [Hornstein et al. \(2011\)](#) note that several search models struggle to generate the observed amount of wage dispersion in the data. Their argument is that on-the-job search is crucial for these models to generate realistic levels of frictional wage dispersion. Our complementary insight is that also the effects of policy, such as the minimum wage, can be amplified in such models.

We also connect our structural search model to empirical studies of wage determination. Whereas several empirical studies document significant dispersion in pay across firms using the original AKM methodology, few studies have provided a formal justification for this framework.⁷ Providing such a microfoundation is important since other papers have stressed that sorting models of labor markets may lead the AKM framework to produce misleading results ([Lentz and Mortensen, 2010](#); [Eeckhout and Kircher, 2011](#); [Lopes de Melo, 2013](#); [Bonhomme et al., 2015](#)). We bridge these two literatures by contributing a tractable model of frictional wage dispersion with heterogeneity in both worker ability and firm productivity that maps directly into the AKM framework. We characterize this mapping and show that the AKM regression framework recovers the underlying structural parameters of the model from the data.

Finally, our focus on the effects of the minimum wage on the earnings distribution complements a long-standing debate in the literature on how the minimum wage affects labor market outcomes. A salient debate in this literature revolves around the employment consequences of the minimum wage ([Card and Krueger, 1994](#); [Neumark and Wascher, 1994](#); [Manning, 2005](#)), with mixed findings but pointing in the direction of small negative employment effects. [DiNardo et al. \(1996\)](#), [Lee \(1999\)](#), [Card and DiNardo \(2002\)](#) argue that a decline in the federal minimum wage in the U.S. in the 1980's explains a significant amount of the increase in earnings inequality during that time. Going against this previous literature's conclusions, [Autor et al. \(2008\)](#) and [Autor et al. \(2016\)](#) argue that nonmarket factors such as the decline in the minimum wage contributed little to the dynamics of U.S. earnings inequality. [Bárány \(2015\)](#) studies a model with complementarity be-

⁷A notable recent exception is [Burdett et al. \(2016\)](#).

tween skill groups in the production technology and endogenous educational investment, giving rise to spill-over effects of the minimum wage through a different mechanism than in our paper. [Harasztosi and Lindner \(2015\)](#) study an increase in the minimum wage in Hungary in 2001 that is of similar size as the one experienced in Brazil 1996–2012, and find that it pushed up wages with only a small negative impact on employment. [Komatsu and Menezes Filho \(2015\)](#) argue empirically that increases in the minimum wage can explain essentially all of the reduction in earnings inequality in Brazil between 2002 and 2013. We complement these papers by showing empirically and using an estimated structural model that the minimum wage in Brazil has had large spill-over effects higher up in the earnings distribution.

Outline. The rest of the paper is structured as follows. Section 2 introduces the three datasets we use to study the evolution of earnings inequality and the role of the minimum wage. Section 3 presents key facts on the decline of earnings inequality in Brazil building on [Alvarez, Benguria, Engbom, and Moser \(2016\)](#). Section 4 provides an institutional history of the minimum wage in Brazil. In Section 5, we describe our structural model of frictional wage dispersion and characterize the effects of a rise in the minimum wage on workers and firms in the economy. Section 6 describes our estimation strategy and the main policy experiment identifying the effects of the observed rise in the minimum wage on the earnings distribution. Section 7 presents quantitative results on the effect of the minimum wage on earnings inequality, on compression throughout the income distribution, and on the productivity-pay gradient across firms. Finally, Section 8 concludes.

2 Data

Our analysis combines data from three separate sources: The first dataset are the Brazilian Household surveys *Pesquisa Nacional por Amostra de Domicílios (PNAD)*, which contain a representative sample of households covering all of Brazil, including workers in the formal and informal sectors. Our second data source consists of an administrative linked employer-employee dataset called *Relação Anual de Informações (RAIS)*, containing annual information from 1996–2012 on earnings and demographic characteristics of formal sector workers as reported by employers. The third dataset is the *Pesquisa Industrial Anual Empresa (PIA)*, which contains information on the revenue and cost

structure of large firms in Brazil’s mining and manufacturing sectors from 1996–2012, and which we merge with the worker-level data contained in RAIS. The following subsections describe each of the three datasets in detail.⁸

2.1 Household survey data (PNAD)

The PNAD household surveys consist of a double-stratified sampling scheme by region and municipality, interviewing a representative of households in Brazil. The survey asks the household head to respond on behalf of all family members and report a rich set of demographic and employment-related questions. In particular, the survey asks a question about whether the respondent holds a legal work permit. We use the answer to this survey question to identify individuals as working in the formal or in the informal sector. Survey questions regarding income and demographics of the respondent household members are comparable to the U.S. March Current Population Survey (CPS). We keep only observations that satisfy our selection criteria and have non-missing observations for labor income, whose variable definition we harmonize across years.⁹ Table 1 presents basic summary statistics on the PNAD data.

2.2 Linked employer-employee data (RAIS)

The RAIS is constructed from a mandatory survey filled annually by all formally registered firms in Brazil. The data collection is administered by the Brazilian Ministry of Labor and Employment, which kindly provided the data for the purposes of this research under a confidentiality agreement. Data collection was initiated in 1986 within a nationally representative set of regions, reaching complete coverage of all employees at tax-registered establishments across all sectors of the economy in 1994. It is common practice for businesses to hire a specialized accountant to help with the completion of the RAIS survey to avoid fines levied on late, incomplete, or inaccurate reports. The data contain a unique, completely anonymized, time-invariant person identifier, which allows us to follow workers over time. It also contains unique, completely anonymized time-invariant establishment and firm IDs that we use to link multiple workers to firms and follow firms over time. We follow our previous work in conducting all analyses at the firm-level.

⁸Appendix ?? also contains summary statistics for PNAD, RAIS, and PIA at a period frequency.

⁹Standardized cleaning procedures are adopted from the Data Zoom suite developed at PUC-Rio and available for replication online at <http://www.econ.puc-rio.br/datazoom/english/index.html>.

Table 1. PNAD summary statistics

		Log earnings		
	(1)	(2)	(3)	(4)
	# Workers	Mean	Std. dev.	Formal share
1996	60,176	6.81	0.98	0.65
1997	64,204	6.79	1.00	0.64
1998	63,016	6.78	0.97	0.64
1999	64,328	6.72	0.95	0.63
2000	-	-	-	-
2001	70,558	6.68	0.95	0.63
2002	72,273	6.66	0.93	0.63
2003	71,959	6.59	0.93	0.64
2004	75,617	6.61	0.91	0.64
2005	78,064	6.64	0.90	0.65
2006	78,627	6.71	0.89	0.66
2007	76,616	6.76	0.87	0.68
2008	76,571	6.80	0.85	0.69
2009	77,037	6.83	0.84	0.70
2010	-	-	-	-
2011	67,884	6.93	0.80	0.73
2012	69,297	6.98	0.80	0.73

Notes: All statistics are for adult male workers of age 18–49. Statistics on earnings are in multiples of the current minimum wage. All numbers reported are for adult male workers. Means are computed by period. The standard deviation is calculated by first demeaning variables by year and then pooling the years within a sub-period. Surveys are not available in years 2000 and 2010.

The dataset contains information on average gross monthly labor earnings including regular salary payments, holiday bonuses, performance-based and commission bonuses, tips, and profit-sharing agreements as well as start and end month of the job. The measure of income adjusts for labor supply by dividing annual earnings by the number of months worked at the job. A worker might have multiple spells in a year if he or she switched employer during the year or worked multiple jobs. We restrict attention to a unique observation per worker-year by choosing the highest-paying among all longest employment spells in any given year. In addition, we observe the age, gender, educational level, and occupation¹⁰ of each worker. On the firm side, we also use sub-sector categories from IBGE, the national statistical institute.¹¹ Our firm size measure is the number of full-time equivalent workers during the reference year.

¹⁰We use occupations from the pre-2003 Classificação Brasileira de Ocupações (CBO) at the two-digit level.

¹¹Both the industry and occupation classification systems changed during the period we study. We use conversion tables provided IBGE to standardize classification between different years and choose categories for both occupations and sectors coarse enough in order to avoid potential biases arising from mechanical changes in the classification system over time.

We exclude individual observations that have either firm IDs or worker IDs reported as invalid as well as data points with missing earnings, dates of employment, educational attainment or age. Together, these basic cleaning procedures drop less than 1% of the original population, indicative of the high quality of the administrative dataset.

Table 2 provides key summary statistics for the RAIS data for six periods spanning 1988-92, 1992-96, 1996-2000, 2000-2004, 2004-08, and 2008-12.¹² All numbers reported in the table are for adult male workers of age 18–49. We make the selection based on gender and age to be consistent with our previous work.¹³ The group of adult males represents 55% of the total dataset in 2000 and their average earnings, educational attainment, and age are largely representative of the overall population.

Table 2. RAIS summary statistics

	(1) # Workers	(2) # Firms	Log earnings	
			(3) Mean	(4) Std. dev.
1996	18.05	0.98	1.32	0.87
1997	18.31	1.06	1.32	0.85
1998	18.65	1.12	1.28	0.85
1999	18.54	1.18	1.25	0.84
2000	19.15	1.22	1.20	0.83
2001	20.45	1.30	1.12	0.83
2002	21.22	1.37	1.06	0.81
2003	21.70	1.42	0.99	0.79
2004	22.78	1.48	0.98	0.78
2005	23.96	1.54	0.94	0.77
2006	25.14	1.61	0.86	0.75
2007	26.58	1.68	0.83	0.74
2008	28.45	1.76	0.83	0.73
2009	29.17	1.84	0.80	0.73
2010	31.01	1.95	0.78	0.71
2011	32.38	2.05	0.81	0.71
2012	33.23	2.13	0.78	0.70

Notes: All statistics are for male workers age 18–49. Statistics on earnings are in multiples of the current minimum wage. All numbers reported are for adult male workers. The standard deviation is calculated by first demeaning variables by year and then pooling the years within a sub-period.

¹²To calculate the standard deviation, we demean the data by year before we pool the years within a subperiod.

¹³Extensive labor supply decisions correlated with schooling choice or the timing of retirement could bias our estimates if we were to include these population subgroups. For similar reasons, we also exclude women to avoid biases caused by job switching decisions motivated by maternal leaves and other motherhood-related labor market movements.

2.3 Firm characteristics data (PIA)

The PIA dataset contains data on firm characteristics from 1996 to 2012. It is constructed from annual surveys filled by firms in the manufacturing and mining sector and collected by the Brazilian Statistics and Geography Institute (*Instituto Brasileiro de Geografia e Estatística*, or IBGE), with whom we have signed a confidentiality agreement. This survey is mandatory for all firms with either more than 30 employees or more than \$300,000 in revenues. As with RAIS, completion of the survey is mandatory and non-compliance is subject to a fine by national authorities. Each firm has a unique, anonymized identifier, which we use to link firm characteristics data from PIA data to worker-level outcomes in the RAIS data. Each firm has a unique, completely anonymized identifier which we use to link the PIA dataset with employee data from RAIS.

The PIA dataset includes a breakdown of operational and non-operational revenues, costs, investment and capital sales, number of employees and payroll. All nominal values are converted to real values using the CPI index provided by the IBGE. Instead of the measure of firm size in the PIA, we prefer our measure of full-time-equivalent employees constructed from the RAIS as it accounts for workers only employed during part of the year. We define operational costs as the cost of raw materials, intermediate inputs, electricity and other utilities, and net revenues as the gross sales value due to operational and non-operational firm activities net of any returns, cancellations, and corrected for changes in inventory.¹⁴ We finally construct value added as the difference between net revenues and intermediate inputs, and value added per worker as value added divided by full-time equivalent workers. This is our main measure of firm productivity.¹⁵

Table 3 shows key summary statistics for the RAIS data for the four periods for which we have firm financial data in the PIA: 1996-2000, 2000-2004, 2004-08, and 2008-12. All results are weighted by the number of full-time equivalent workers employed by the firm.

3 Facts about Brazil's inequality decline

Why does the rise of the minimum wage seem like a promising candidate explanation behind Brazil's earnings inequality decline between 1996 and 2012? We present three key facts on the

¹⁴We have explored alternative revenue definitions such as only restricting attention to operational revenues or excluding certain types of non-operational revenues. Such robustness checks yield very similar results to what we report below.

¹⁵We have also constructed alternative measures of firm productivity by cleaning value added per worker off industry-year effects and some measures of worker skill.

Table 3. PIA Summary Statistics

	(1) # Firm-years	Log revenues per worker		Log value added per worker	
		(2) Mean	(3) S.d.	(4) Mean	(5) S.d.
1996	21,840	11.83	1.00	11.15	1.08
1997	21,022	11.86	1.03	11.15	1.11
1998	22,096	11.88	1.09	11.17	1.19
1999	22,771	12.01	1.16	11.29	1.27
2000	22,751	12.00	1.19	11.22	1.30
2001	24,920	12.01	1.24	11.23	1.32
2002	26,418	12.02	1.30	11.26	1.39
2003	27,853	11.96	1.31	11.18	1.37
2004	28,708	12.00	1.32	11.21	1.35
2005	30,628	11.94	1.30	11.16	1.33
2006	31,962	11.94	1.28	11.18	1.32
2007	31,808	11.97	1.28	11.21	1.31
2008	33,349	12.01	1.27	11.26	1.30
2009	34,200	12.01	1.23	11.31	1.27
2010	34,650	12.03	1.22	11.32	1.25
2011	36,773	12.06	1.20	11.34	1.23
2012	37,858	12.07	1.18	11.36	1.20

Notes: Sample includes all firms covered by the PIA dataset in the mining and manufacturing sectors. The number of firm-years and number of unique firms are reported in thousands. All means and standard deviations are weighted by the number of employees. The standard deviation is calculated by first demeaning variables by year and then pooling the years within a sub-period.

evolution of earnings inequality in Brazil over this period that any explanation plausibly needs to be consistent with.

Our motivating observation is that earnings inequality has declined rapidly in Brazil (Alvarez et al., 2016). Figure 1 plots the evolution of the variance of log earnings of prime-age male workers in the formal sector between 1996 and 2012.¹⁶ The data show a steady decline in the variance of log earnings by 26 log points or 35 percent, from 0.76 to 0.49, over the period. To put this evolution into context, in the U.S. the variance of log earnings for adult male workers increased by approximately eight log points over the same period (Heathcote et al., 2010; Kopczuk and Saez, 2010).

We now present three facts characterizing Brazil's inequality decline between 1996 and 2012.

Fact 1. *The inequality decline is due to bottom-driven but wide-spread real wage growth.*

¹⁶ All statistics are computed for the population of male formal sector workers aged 18–49 covered by the RAIS data. We later discuss trends in Brazil's informal sector, which concurrently declined in size, using the PNAD household survey data.

Figure 1. Decline in the variance of log earnings in Brazil, 1996–2012

While overall inequality fell rapidly, some parts of the earnings distribution compressed more rapidly than others. Panel (a) of Figure 2 shows the real earnings evolution of various percentiles of the distribution from 1996–2012, normalized to zero in the initial year. All earnings percentiles increased in real levels, with the 90th percentile (long dashed teal line with hollow circles) growing by 50 log points. But lower earnings percentiles experienced relatively higher earnings growth, with the tenth percentile (dashed red line with filled squares) growing by 120 log points over this period. It is this 70 log points relative real earnings growth at the bottom of the distribution that we seek to explain.

Panel (b) of Figure 2 summarizes the implied dynamics of top- and bottom-inequality by plotting the log 90–50 percentile ratio (solid blue line with filled circles) and the log 50–10 percentile ratio (dashed red line with filled squares) of the earnings distribution. Both measures decline significantly but the log 50–10 percentile ratio markedly more so than the log 90–50 percentile ratio. Specifically, the log 50–10 percentile ratio declined by 38 log points while the log 90–50 percentile ratio declined by 19 log points at the same time. Indeed, earnings compressed little above the 75th percentile, with the very top of the distribution actually diverging slightly over the period.

Fact 2. *Low-income regions saw more pronounced compression in the lower half of the earnings distribution.*

To investigate the sources of Brazil’s bottom-driven inequality decline, we track different parts

Figure 2. Normalized evolution of earnings percentiles, 1996–2012

of the earnings distribution across 27 states over time between 1996 and 2012. In a seminal paper quantifying the contribution of the falling minimum wage towards earnings inequality in the U.S., [Lee \(1999\)](#) suggests a theoretical and empirical framework, which we adopt to the Brazilian context. Following that methodology, we compute percentile ratios relative to median earnings (e.g. the P50-P10 ratio, P90-P50 ratio etc.) across states of the Brazilian federation at an annual level.¹⁷ As the federal minimum wage increases over time in Brazil, we see a rise in the minimum-to-median earnings ratio—the “effective minimum wage”, in the language of [Lee \(1999\)](#). As the effective minimum wage increases, we track lower tail inequality, measured by the P50-P10 ratio, and upper tail inequality, measured by the P90-P50 ratio, over time.

Two points relating to the results, plotted in Figure 3, are noteworthy: First, states that were more binding relative to the federal minimum wage in 1996, i.e. those with lower initial median income, subsequently experienced a more pronounced decline in lower-tail (but not upper-tail) inequality. In fact, the most binding states see lower-tail inequality decline essentially one-for-one with the minimum wage, suggesting that the minimum wage carries along low-paid workers, conditional on remaining employed in those regions.¹⁸

Second, lower-tail inequality also declines, although less than one-for-one, in states for which the minimum wage is initially less binding. This observation is consistent with the minimum wage

¹⁷For this exercise like for our regression analysis below, we focus on individuals’ highest-paid among all longest-lasting employment spells during a year to compute monthly earnings on the job.

¹⁸As [Lee \(1999\)](#) notes, it is important to control for non-employment changes over time in this exercise. Indeed, we find that unemployment levels in 2012 were at approximately the same levels as in 1996 and that the informal sector shrank in size over this period, thus alleviating any such concerns.

boosting wages at the bottom of the earnings distribution even for workers not directly affected by the minimum wage. Consistent with the notion of “spill-over effects” of the minimum wage reaching up above the minimum but fading out towards the top of the distribution, we find that upper-tail inequality, measured by the P90-P50 ratio, sees little to no change in response to the effective minimum wage increasing.

Figure 3. Inequality evolution by state and initial bindingness of the minimum wage

We formalize these results in a regression framework based on [Lee \(1999\)](#) and further developed in a recent paper by [Autor et al. \(2016\)](#), by regressing various earnings percentile ratios on the effective bindingness of the minimum wage along with various sets of controls. Specifically, we regress the log earnings of percentile p relative to the median in state s in year t on a polynomial in the effective bindingness of the minimum wage, state fixed effects, state-specific time trends, and year effects:

$$w_{st}(p) - w_{st}(50) = \sum_{n=1}^N \beta_n(p) \left[w_t^{min} - w_{st}(50) \right]^N + \sigma_{s,1}(p) + \sigma_{s,2}(p) \times t + \gamma_t(p) + \varepsilon_{st}(p) \quad (1)$$

where N is the order of the polynomial in the effective bindingness. After estimating equation (1), we compute the marginal effects of minimum wage as

$$\rho_p \equiv \frac{\partial [w_{st}(p) - w_{st}(50)]}{\partial [w_t^{min} - w_{st}(50)]} = \sum_{n=1}^N (n) \beta_n(p) \left[w_t^{min} - w_{st}(50) \right]^{n-1}$$

Table 4 shows the results of this exercise for regressions of polynomial order two.¹⁹ While all specifications yield qualitatively consistent results, our preferred specification is column (3), which controls for both state fixed effects and state trends. Near the bottom of the earnings distribution, the minimum wage compresses the earnings distribution one-for-one, as seen by the coefficient on the P50-P10 having a point estimate of 0.975, which is statistically indistinguishable from 1. More interestingly, we also find significant spill-over effects of the minimum wage at the tenth percentile (significant point estimate of 0.799), at the 25th percentile (significant point estimate of 0.468) and above. But these spill-over effects die out towards the 75th percentile, suggesting that the minimum wage lead to compression up to but not including the top quartile of the earnings distribution.

Table 4. Marginal effects of minimum wage throughout earnings distribution (ρ_p)

	(1)	(2)	(3)
$p = 5$	0.670*** (0.050)	0.926*** (0.074)	0.975*** (0.064)
$p = 10$	0.485*** (0.060)	0.804*** (0.075)	0.799*** (0.064)
$p = 25$	0.217*** (0.042)	0.398*** (0.056)	0.468*** (0.059)
$p = 75$	-0.092 (0.075)	0.037 (0.092)	-0.026 (0.100)
$p = 90$	0.025 (0.126)	0.220 (0.114)	0.182 (0.143)
Levels/differenced	Levels	Levels	Differenced
Polynomial degree	$N = 2$	$N = 2$	$N = 2$
Year fixed effects	Yes	Yes	Yes
State fixed effects	No	Yes	Yes
State trends	No	Yes	Yes

* = significant at the 10% level, ** = 5%, *** = 1%. Time period is 1988–2012.

Thus, we conclude that lower-tail inequality fell more rapidly in lower income regions, and that compression as a result of minimum wage increases over time occurred primarily in the lower half of the earnings distribution.

Fact 3. *Essentially all of the explained inequality decline is due to a compression in the returns to firm and worker characteristics in pay.*

We dissect the decline in earnings inequality in Brazil by reviewing key results from an empirical earnings decomposition presented in Alvarez et al. (2016). In that work, we classify the overall inequality evolution into changes in firm pay components and changes in worker pay heterogeneity, following an estimation methodology pioneered by AKM. Specifically, we estimate a two-way

¹⁹The distinction between polynomial orders one and two matters, but we tried higher order polynomials without obtaining significantly different results to those presented below.

fixed effect regression of log monthly earnings on a large set of worker effects, firm effects and year dummies in five-year sub-periods:

$$\log y_{ijt} = \alpha_i^p + \alpha_{J(i,t)}^p + \gamma_t + \varepsilon_{it} \quad (2)$$

for $t \in p = \{t_1, \dots, t_5\}$ and where α_i^p denotes the individual fixed effect of worker i in period p , $\alpha_{J(i,t)}^p$ denotes the firm effect of the employer of worker i at year t , γ_t is a year dummy, and ε_{it} is an error term that satisfies the strict exogeneity condition $\mathbb{E}[\varepsilon_{it} | i, t, J(i, t)] = 0$.²⁰

Table 5 presents results from the above regression. In particular, we compute and report the variance of the predicted value due to each component from the AKM framework in equation (2). The variance of firm effects falls from 17 log points in 1996–2000 to eight log points in 2008–2012, which constitutes 45 percent of the overall inequality decline over the period. Similarly, the variance of worker effects falls from 36 log points in 1996–2000 to 31 log points in 2008–2012, making up 24 percent of the overall decline. Thus, more equal pay across firms explains a disproportionate share of Brazil’s inequality decline.

Table 5. AKM variance decomposition between periods

	(1) 1996-2000	(2) 2008-2012	(3) Change
Total variance of log earnings	0.72 (100%)	0.52 (100%)	-0.20 (100%)
Variance of worker fixed effects	0.36 (50%)	0.31 (60%)	-0.05 (24%)
Variance of firm fixed effects	0.17 (23%)	0.08 (15%)	-0.09 (45%)
2×Covariance b/w workers and firms	0.14 (20%)	0.10 (20%)	-0.04 (22%)
Residual variance	0.06 (8%)	0.04 (7%)	-0.02 (10%)
Worker years	90.2	151.0	
R^2	0.92	0.93	

Note: Cells contain variance (share) explained by each component. Year dummies are omitted but account for a negligible share of the overall variation. The “Covariance” term reports two times the covariance between worker effects and firm effects from the AKM estimation. Number of worker years is in millions.

In a second step, we relate the overall decline in the variance of firm effects and worker effects

²⁰ Alvarez et al. (2016) present a range of robustness checks for this specification and conclude that the model fits well the Brazilian data during this period.

to observable firm and worker characteristics:

$$\hat{\alpha}_{J(i,t)}^p = \zeta_p \text{VAPW}_j + \sum_s \zeta_s \mathbb{1}(\text{sector}_{J(i,t)} = s) + \sum_r \zeta_r \mathbb{1}(\text{state}_{J(i,t)} = r) + \eta_j$$

$$\hat{\alpha}_i^p = \sum_a \zeta_a \mathbb{1}(\text{age}_i = a) + \sum_e \zeta_e \mathbb{1}(\text{edu}_i = e) + \eta_i$$

where VAPW_j denotes log value added per worker at the firm-level, $\left\{ \mathbb{1}(\text{sector}_{J(i,t)} = s) \right\}_s$ is a set of 28 sector dummies, $\left\{ \mathbb{1}(\text{state}_{J(i,t)} = r) \right\}_r$ denotes 27 state dummies, while $\left\{ \mathbb{1}(\text{age}_i = a) \right\}_a$ and $\left\{ \mathbb{1}(\text{edu}_i = e) \right\}_e$ represent indicators for four age groups and four education groups, respectively.

Based on these regressions, we attribute the explained decline in the variance of the left-hand side variables of these equations to two potential explanations: compression in the distribution of underlying characteristics (holding fixed estimated loadings) versus compression in the loadings (holding fixed initial distributions).

To address the first potential explanation, we find that pay-relevant firm characteristics—including value added per worker, firm size, and export intensity—all have become more dispersed over this period (Alvarez et al., 2016). Thus, in spite of greater underlying inequality, Brazilian firms offer more equal pay over time. Similarly, on the worker side we find that changes in the distribution of pay-relevant worker characteristics—such as age and education—have contributed little to the declining dispersion of worker pay components.

In line with the second potential explanation, we confirm that compression in the returns to pay-relevant firm and worker characteristics is driving a large decline in firm pay and worker pay heterogeneity, respectively. To see this on the firm side, Table 6 illustrates the declining productivity-pay gradient among manufacturing and mining firms covered in the PIA data. Between the two periods 1996-2000 and 2008-2012, the correlation coefficient between firm effects in pay and value added per worker dropped from 0.210 to 0.112, implying a five log points reduction in the variance of log earnings while keeping fixed the initial productivity distribution (which widened over this period). We conclude that a weakening firm productivity-pay gradient accounts for almost all of the explained decline in the variance of firm effects, constituting over 25 percent of the overall inequality decline.

Another five log points or 24 percent of the decline in the variance of log earnings are due to compression in estimated worker effects in the above AKM framework. Analogously to our firm-level analysis, we predict the variance explained by age and education in the data and ask

Table 6. Regression of firm pay component on productivity

	(1) 1996–2000	(2) 2008–2012	(3) Change
PANEL A. REGRESSION RESULTS			
Value added p.w.	0.210	0.112	-0.098
# worker years	16.6	26.3	
R^2	0.711	0.657	
PANEL B. VARIANCE DECOMPOSITION			
Explained variance	0.10	0.05	-0.05
—due to returns			-0.05
—due to composition			0.01

Note: Dependent variable is AKM estimate of firm effect on wages, $\hat{a}_{j(i,t)}^p$, controlling for state and industry indicators. Explained variance holds R^2 fixed in 1996–2000. Number of worker years is in millions.

whether changes in the explained variance of worker effects derives from compression in the distribution of or returns to those characteristics. Table 7 shows the result from this exercise. Estimated coefficients on both age and education uniformly decline over time, explaining a 3.1 log points decline in the variance of log earnings over the period. As on the firm side, all of this decline is driven by a compression in the returns to the characteristics rather than due to changes in the underlying composition of workers.²¹

Summary. Summarizing Facts 1–3 above, Brazil experienced a large decline in earnings inequality between 1996–2012; the decline was bottom-driven; lower-tail inequality declined most rapidly in lower income states and industries; and the decline was primarily due to a compression in relative returns to firm productivity and worker characteristics in pay. The above facts narrow down the set of possible explanations for the sharp fall in earnings dispersion. We will next argue that the rise in the minimum wage over this period will naturally explain these facts.

4 The minimum wage in Brazil

Can the rise of the minimum wage explain the large decline in earnings inequality in general, and the three facts discussed in Section 3 in particular? To answer, this question, we provide institutional context and a description of the evolution of the minimum wage in Brazil.

²¹One may suspect that also the returns to unmeasured ability have declined over this period. In this case, our results should be interpreted as a lower bound on the true decline explained by a compression in returns.

Table 7. Regression of estimated worker effects on worker characteristics

	(1)	(2)	(3)
	1996–2000	2008–2012	Change
PANEL A. REGRESSION RESULTS			
Age 25–29	0.20	0.16	-0.04
Age 30–39	0.39	0.30	-0.09
Age 40–49	0.52	0.42	-0.10
Middle school	0.21	0.11	-0.10
High school	0.61	0.27	-0.34
College or more	1.21	1.10	-0.11
# worker years	90.2	151.0	
R^2	0.34	0.37	
PANEL B. VARIANCE DECOMPOSITION			
Explained variance	0.11	0.08	-0.03
—due to returns			-0.03
—due to composition			0.01

Note: Dependent variable is the estimated worker effect a_i . Number of workers in millions. Age estimates are relative to “age 18–24” category. Education estimates are relative to “less than middle school (<7 years)” category. Number of worker years is in millions.

4.1 History

The minimum wage in Brazil is primarily a federal institution with only minor adjustments for regional price level differences. It was institutionalized as Decree-Law No. 2162 in 1940 and consolidated in 1943 under new labor laws (*Consolidação das Leis do Trabalho*, or *CLT*).²² While the minimum wage was initially region-specific and not automatically adjusted to inflation or even legally enforced, it underwent several reforms under different political regimes between the 1940s and 1984, when it was unified across regions.

Leading up to and during Brazil’s hyperinflationary period from 1980–1994, the minimum wage was adjusted first annually and later at monthly intervals according to a formula based on realized productivity growth and inflation as well as expected future inflation. Yet, due to forecasting errors in the price level during these turbulent times, the minimum wage lost over a third in real value. Following several failed stabilization plans, the *Plano Real* in 1994 stabilized the monetary system by pegging the local currency to the U.S. dollar (it was allowed to float again

²²The original law was based in parts on Mussolini’s Carta del Lavoro in Italy.

starting in 1999).²³ With the monetary stabilization of 1994 and the new president Fernando Henrique Cardoso of the centrist Brazilian Social Democracy Party taking office in 1995, the minimum wage became a renewed policy focus.

Nowadays, the minimum wage acts as a floor on monthly earnings for every formally registered worker. The Brazilian Ministry of Labor (*Ministério do Trabalho e Emprego*, or MTE) heads a compliance unit, which audits businesses through on-site visits and surveying local employees. Yet, according to official reports as well as information from our household and administrative data sources, compliance is less than perfect. While overall compliance is thought to be good in the formal sector, the minimum wage is plausibly less binding in Brazil's sizable informal economy.²⁴

4.2 Evolution of the minimum wage

Between 1988 and 1996, the real minimum wage declined and experienced significant volatility as a result of hyperinflation. Then between January 1996 and December 2012, the real minimum wage grew by a total of 119 percent, reaching a value of 622 BRL (410 PPP-adjusted USD) per month by the end of the period. To put these numbers into context, the minimum wage as a fraction of median earnings increased from around 34 percent in 1996 to 60 percent in 2012. Over the same period, average labor productivity in manufacturing and mining increased by 16.6 percent; hence the ratio of the minimum wage to average labor productivity increased by 56.3 percent over this period.

Figure 4 summarizes the evolution of the variance of log earnings (in blue) and also annual averages of the real minimum wage (in red) between 1988 and 2012.²⁵ Suggestive of the minimum wage being related to the evolution of earnings inequality, we see that the two time series approximately mirror each other, with the correlation between the two being -0.82 in levels and -0.55 in Hodrick-Prescott (HP) filter cycles over the period.

The visual compression in the lower tail of the earnings distribution between 1996–2012 is striking. Figure 5 shows a histogram of the earnings distribution for 1996 in panel (a) and for 2012

²³See Garcia et al. (2014) for a comprehensive summary of Brazil's inflation experience and the effects of the various stabilization plans.

²⁴We will return to the distinction between the effects of the minimum wage on Brazil's formal and informal sectors in Section ?? of the current paper.

²⁵For comparison, Appendix A.3 also shows 3-month running averages of the real minimum wage over the period.

Figure 4. Evolution of earnings inequality versus the minimum wage, 1988–2012

in panel (b).²⁶ Inspecting the graphs we note that the distribution becomes heavily compressed in the left tail, leading to increased positive skewness over time.

Figure 5. Histogram of the earnings distribution

A third point worth noting is that the share of workers earning the minimum wage is surprisingly small given the large increase in the minimum wage between 1996 and 2012. Figure 6 shows

²⁶We include annual histograms over the entire period 1996–2012 in Figure 22 of the Appendix.

two measures of the bindingness of the minimum wage, namely the share of workers earning exactly the legal minimum wage, the share at or below the minimum wage, and the share within a five percent band around the minimum wage. While the distribution shows some bunching at the bottom in both years, the mass of workers earning exactly the minimum wage is in the range of two percent and stable over time. The share of workers earning up to the minimum wage is slightly higher but grows little in absolute terms over time, going from three percent in 1996 up to five percent in 2012. Even using our most generous measure, which computes the share of workers within a five percent band around the minimum wage to account for measurement error, never exceeds seven percent throughout this period. Indeed, our model will replicate the fact that a negligible number of people will be binding exactly at the minimum wage at any point in time.

Figure 6. Bindingness of the minimum wage, 1996–2012

4.3 Evaluating effects of the minimum wage through a structural model

While the joint evolution of the minimum wage and earnings inequality between 1988 and 2012 suggest that the two trends might be related, it remains an open question whether their relationship is causal. Importantly, one may note that the direct effect of the minimum wage is bounded above by the fraction of workers between the original and the new level of the minimum wage.²⁷

²⁷ An even more critical view would suggest that the share of people affected by the minimum wage is restricted to the share of workers working exactly at the new minimum wage, after the increase. For this to be true, one would need to rationalize disproportionately fast productivity growth among workers with the lowest earnings, which we

In spite of the large increase in real levels, a back-of-the-envelope calculations shows that these direct effects fall short of explaining the massive decline in earnings inequality over the period, as documented in the beginning of Section 3. Furthermore, the direct effects of the minimum wage could hardly speak to either the global compression of the earnings distribution documented in Fact ??, nor could they quantitatively explain the documented drop in the productivity-pay gradient across firms as well as the lower returns to worker characteristics noted in Facts ??–??.

A simplistic view of the minimum wage would thus conclude that its effects are limited to a small population subgroup and its effects have difficulty explaining the three facts from Section 3. Hence, in order for the minimum wage to be a promising candidate explanation, its effects need to extend beyond the direct impact on workers earning exactly the minimum wage.

Contrary to this simplistic view, a strand of the labor economics literature has suggested that the minimum wage might lead to spillover effects further up in the earnings distribution. Theories of such indirect effects of the minimum wage go back to at least [Burdett and Mortensen, 1998](#). In a frictional labor market, monopsonistic competition among firms for workers would lead a minimum wage hike to affect wages of workers strictly above the minimum wage. In such an environment, is possible that an increase in the minimum wage ripples through the earnings distribution through such equilibrium effects. Recently, ? took up this debate empirically in arguing that the magnitude of such spillover effects is indistinguishable from measurement error in the data in the case of the U.S. labor market. To take up this debate and evaluate the importance of such channels for the case of Brazil, we proceed to build and estimate a structural equilibrium model of the Brazilian labor market. We then proceed to use the estimated model to quantify the degree of spillover effects from minimum wage increases over the period.

5 Model

We investigate the effects of a rising minimum wage on the earnings distribution in an equilibrium search model matching key characteristics of the Brazilian labor market. Our model rationalizes the fact that identical workers experience large pay differences across employers in a frictional labor market populated by monopsonistic firms. A key prediction of our model, which we confirm in the data, is that job-to-job mobility leads workers to climb a “job ladder” by gradually moving

view as broadly incompatible with widely held beliefs that technical change over this period was characterized as high skill-biased.

to better-paying employers. As a result, firms compete for workers by setting wages strategically relative to one another and in reference to the minimum wage.

5.1 Environment

Time is continuous and we restrict attention to a stationary environment without aggregate shocks. A unit mass of heterogeneous workers and a positive mass of heterogeneous firms meet in a labor market subject to search frictions. In the spirit of [van den Berg and Ridder \(1998\)](#), search is segmented in the sense that different worker types search in separate markets while firms decide how many vacancies to create in each market. Search is also random in the sense that within each market workers cannot direct their search towards specific firms. The assumption of random search in segmented markets facilitates analytical tractability of the model while capturing the idea that the process of filling a vacancy is costly and firms can condition job offers on certain worker attributes (e.g. education).

5.2 Workers

Workers differ in their permanent ability level θ , which is time-invariant and distributed according to H over support $[\underline{\theta}, \bar{\theta}]$. They are infinitely-lived and value a stream of expected consumption discounted at rate ρ .

Search occurs in labor markets segmented by worker types, both from non-employment and while employed. Labor market frictions imply that workers' employment status and job attributes are stochastic. Let λ_θ^u denote the instantaneous rate at which a non-employed worker receives a job offer and let λ_θ^e be the arrival rate for an employed worker. A job offer is an opportunity to work for a wage w drawn from distribution $F_\theta(w)$ with support $[\underline{w}_\theta, \bar{w}_\theta]$. Although a worker treats job finding rates and the distribution of job offers as given, they will be determined endogenously in equilibrium through firms' optimal job creation and wage posting decisions. A job is terminated either endogenously when workers move towards a preferred job opportunity, or exogenously with probability δ_θ , in which case workers flow back to the non-employed pool.

Let W_θ denote the value function of a non-employed worker of ability θ and $S_\theta(w)$ the value function of the same worker employed at wage w . These value functions satisfy the Bellman

equations

$$\rho W_\theta = b_\theta + \lambda_\theta^u \int_{\underline{w}_\theta}^{\bar{w}_\theta} \max \{S_\theta(w) - W_\theta, 0\} dF_\theta(w)$$

and

$$\rho S_\theta(w) = w + \lambda_\theta^e \int_w^{\bar{w}_\theta} [S_\theta(w') - S_\theta(w)] dF_\theta(w') + \delta_\theta [W_\theta - S_\theta(w)]$$

The value function S_θ is strictly increasing in w , and hence the optimal strategy of a non-employed worker is characterized by a reservation wage ϕ_θ . A non-employed worker accepts all wage offers above ϕ_θ and rejects offers below it. Following [Burdett and Mortensen \(1998\)](#), one can show that the reservation wage ϕ_θ is implicitly defined by

$$\phi_\theta = b_\theta + (\lambda_\theta^u - \lambda_\theta^e) \int_{\phi_\theta}^{\bar{w}_\theta} \frac{1 - F_\theta(w)}{\rho + \delta_\theta + \lambda_\theta^e (1 - F_\theta(w))} dw$$

The interpretation is straightforward: a worker requires to be paid at least his flow value of unemployment, plus the option value forgone when leaving unemployment. That option value arises due to a potentially different job finding rate in employment versus unemployment. The lowest wage at which a worker of type θ can be employed is thus $\max \{\phi_\theta, w^{\min}\}$, and we refer to $w^{\min} > \phi_\theta$ as a binding minimum wage in market θ .

Denote by u_θ the unemployment rate in market θ . In the stationary equilibrium, a standard flow balance equation solved for the stationary solution implies that

$$u_\theta = \frac{\delta_\theta}{\delta_\theta + \lambda_\theta^u} \quad (3)$$

Let G_θ denote the wage distribution in market θ . Because employed workers gradually move to better jobs, G_θ in general differs from the offer distribution F_θ . Writing down the law of motion for G_θ , and solving for the stationary solution

$$G_\theta(w) = \frac{F_\theta(w)}{1 + \kappa_\theta (1 - F_\theta(w))} \quad (4)$$

where $\kappa_\theta \equiv \frac{\lambda_\theta^e}{\delta_\theta}$ governs the relative speed of climbing up the job ladder.

5.3 Firms

Firms are characterized by a constant productivity level p , drawn from distribution Γ_0 with continuous support $P = [p_0, \bar{p}]$. Firms produce output by combining workers of different ability levels using a linear production technology. Together with the assumption of perfect segmentation of labor markets by ability types, the assumption of a linear production technology improves tractability because it abstracts from interactions across θ markets. Letting l_θ denote the number of employees from market θ , flow output of a firm with productivity p is

$$y(p, \{l_\theta\}_{\theta \in \Theta}) = p \int_{\theta \in \Theta} \theta l_\theta d\theta$$

A firm attracts workers of type θ by posting job openings in that market, v_θ , subject to a strictly convex, increasing flow cost $c_\theta(v_\theta)$. A job opening is a promise to pay a wage, w_θ , for the remainder of the match. Firms are assumed to be able to commit to the posted wage.

In equilibrium, the number of jobs and the wage a firm posts jointly determine the amount of workers it attracts, $l_\theta = l_\theta(w_\theta, v_\theta)$. In choosing what wage to post, a firm trades off two forces. On the one hand, it attracts a greater mass of workers per posted vacancy by offering a higher wage relative to its competitors, and it retains a larger fraction of its workforce, both of which result in higher output. On the other hand, a higher wage reduces its profits per employed worker. Given its positioning in the pay rank, a higher number of vacancies increases the number of workers a firm attracts, but is associated with higher recruiting costs.

Because workers of different ability are perfect substitutes, the firm maximizes profits in each labor market separately. The problem faced by a firm with productivity p in market θ is to choose a mass of jobs to create and a wage to associate with those jobs in order to maximize flow output²⁸

$$\max_{w_\theta \geq w^{min}, v_\theta} (p\theta - w_\theta) l_\theta(w_\theta, v_\theta) - c_\theta(v_\theta)$$

To attract workers and make profits, a firm needs to post at least the worker's outside option, ϕ_θ . As a result, only firms with productivity above $\underline{p}_\theta \equiv \max\{w^{min}, \phi_\theta\} / \theta$ are active in that

²⁸We follow [Burdett and Mortensen \(1998\)](#) in assuming the limiting case of $r/\lambda_\theta^u \rightarrow 0$, rendering the maximization of flow output the appropriate objective.

market. Denote by Γ_θ the distribution of active firms in market θ ,

$$\Gamma_\theta(p) = \frac{\Gamma_0(p) - \Gamma_0(\underline{p}_\theta)}{1 - \Gamma_0(\underline{p}_\theta)}$$

Denote by $v_\theta(p)$ the optimal vacancy posting rule that solves the firm's problem in market θ , and by $w_\theta(p)$ the optimal wage posting rule. The total mass of jobs in market θ is

$$V_\theta = \int_{\underline{p}_\theta}^{\bar{p}} v_\theta(p) d\Gamma_\theta(p) \quad (5)$$

Taken for given for now that the optimal wage posting rule is monotonic in productivity, we can write the wage offer distribution as

$$F_\theta(w_\theta(p)) = \int_{\underline{p}_\theta}^p \frac{v_\theta(\tilde{p})}{V_\theta} d\Gamma_\theta(\tilde{p}) \quad (6)$$

We show below that wages are indeed increasing in productivity.

5.4 Matching

We assume that employed workers search with efficiency s_θ relative to unemployed workers and that an aggregate matching function brings together searching workers and firms. Following much of the literature, we assume that the matching function is on the Cobb-Douglas form, $M[u + s(1 - u), V] = \chi [u + s(1 - u)]^{1-\alpha} V^\alpha$, where α governs the elasticity of matches with respect to vacancies and χ is matching efficiency. Letting q_θ denote the worker finding rate of open jobs, we can express the respective finding rates as

$$\lambda_\theta^u = \chi \left(\frac{V_\theta}{u_\theta + s_\theta(1 - u_\theta)} \right)^\alpha, \quad \lambda_\theta^e = s_\theta \lambda_\theta^u, \quad \text{and} \quad q_\theta = \chi \left(\frac{u_\theta + s_\theta(1 - u_\theta)}{V_\theta} \right)^{1-\alpha} \quad (7)$$

5.5 Equilibrium

Before we are ready to define an equilibrium in our economy, we need to characterize the number of workers that a firm obtains if it posts v_θ jobs paying wage w_θ in market θ . The following law of

motion characterizes the evolution of firm size,

$$\Delta l_\theta(w, v) = -\delta_\theta l_\theta(w, v) - s_\theta \lambda_\theta^u (1 - F_\theta(w)) l_\theta(w, v) + v q_\theta \left[\frac{u_\theta}{u_\theta + (1 - u_\theta) s_\theta} + \frac{(1 - u_\theta) s_\theta}{u_\theta + (1 - u_\theta) s_\theta} G_\theta(w) \right]$$

A fraction δ_θ of a firm's employees exit to unemployment and a fraction $s_\theta \lambda_\theta^u (1 - F_\theta(w))$ move on to better employers. A vacancy meets with a worker with probability q_θ , who is unemployed with probability $\frac{u_\theta}{u_\theta + (1 - u_\theta) s_\theta}$ and employed with complementary probability. All unemployed workers accept the offer, while a fraction $G_\theta(w)$ of employed workers accept the offer. Solving for the stationary solution,

$$l_\theta(w, v) = \left(\frac{1}{\delta_\theta + s_\theta \lambda_\theta^u (1 - F_\theta(w))} \right)^2 \frac{v_\theta}{V_\theta} u_\theta \lambda_\theta^u (\delta_\theta + s_\theta \lambda_\theta^u) \quad (8)$$

Definition 1. A stationary search equilibrium is a set of reservation policies $\{\phi_\theta\}_{\theta \in \Theta}$; wage policies and job creation policies $\{w_\theta(p), v_\theta(p)\}_{\theta \in \Theta}$; wage offer distributions $\{F_\theta(w)\}_{\theta \in \Theta}$; firm sizes $\{l_\theta(w, v)\}_{\theta \in \Theta}$; unemployment rates $\{u_\theta\}_{\theta \in \Theta}$; total jobs created, $\{V_\theta\}_{\theta \in \Theta}$; and transition rates $\{\lambda_\theta^u, \lambda_\theta^e\}_{\theta \in \Theta}$ such that:

1. Given the labor market transition rates and the offer distribution, the reservation policies solve each worker type's problem;
2. Given $l_\theta(w, v)$, wage policies and job creation policies solve the firm's problem in each market;
3. The unemployment rates are consistent with equation (3), total vacancies are the sum of individual firms' job creation decisions as in equation (5), and the transition rates are determined by the aggregate matching function and relative search intensity in equation (7);
4. The mapping from wage policies and job creation policies into firm sizes is given by equation (8);
5. The wage offer distributions are given by equation (6).

Define the piece rate, $\tilde{w}_\theta$, such that $w_\theta = \theta \tilde{w}_\theta$. Using the stationary mapping (8) from wages and job offers to firm size, we can re-state the problem of firm p in market θ as

$$\max_{v, \tilde{w}} \left\{ T_\theta v (p - \tilde{w}) \left(\frac{1}{\delta_\theta + s_\theta \lambda_\theta^u (1 - F_\theta(\tilde{w}))} \right)^2 - c_\theta(v) \right\}$$

subject to $\tilde{w} \geq \max \{w^{\min}, \phi_\theta\} / \theta$, where

$$T_\theta = \theta \frac{u_\theta \lambda_\theta^u (\delta_\theta + s_\theta \lambda_\theta^u)}{V_\theta}$$

The first-order condition with respect to vacancies is

$$c'(v_\theta(p)) = T_\theta (p - \tilde{w}) \left(\frac{1}{\delta_\theta + s_\theta \lambda_\theta^u (1 - F_\theta(\tilde{w}))} \right)^2$$

Since profits are increasing in productivity and c_θ is strictly convex, it follows that $v'_\theta(p) > 0$. That is, more productive employers create more jobs.

The first-order condition with respect to piece rates is

$$1 = (p - \tilde{w}_\theta(p)) \frac{2s_\theta \lambda_\theta^u f_\theta(\tilde{w}_\theta(p))}{\delta_\theta + s_\theta \lambda_\theta^u (1 - F_\theta(\tilde{w}_\theta(p)))}$$

Using a similar single-crossing argument as in [Burdett and Mortensen \(1998\)](#), we can show that $\tilde{w}_\theta(p)$ is strictly increasing in productivity.

Appendix C outlines the algorithm we use to numerically solve the problem based on these first-order conditions.

5.6 Theoretical results

Before numerically solving and estimating the model, it is instructive to consider a simple version to illustrate the mechanics. To this end, we abstract for now from vacancy creation. In this case, a straight-forward extension of the framework in [Burdett and Mortensen \(1998\)](#) to our environment with worker heterogeneity yields that the unique equilibrium wage offered by a firm p is

$$w(p, \theta) = \theta p - \theta \int_{\underline{p}_\theta}^p \left[\frac{1 - \Gamma(\underline{p}_\theta) + \kappa_\theta (1 - \Gamma(p))}{1 - \Gamma(\underline{p}_\theta) + \kappa_\theta (1 - \Gamma(x))} \right]^2 dx \quad (9)$$

Our model nests as a special case the environment without a binding minimum wage and p_θ and κ_θ shared across θ markets. In this case, wages in our model satisfy exactly the log additive wage specification of [Abowd et al. \(1999\)](#). That is, a firm pays different θ workers a constant multiple of their underlying worker ability, so that log wages are additively separable into a worker and a firm component:

$$\log w(p, \theta) = \underbrace{\log \theta}_{\text{"worker effect"}} + \underbrace{\log \tilde{w}(p)}_{\text{"firm effect"}} \quad (10)$$

With a binding minimum wage, the second term in equation (9) depends on θ , perturbing the exact decomposition in equation (10). We now characterize how exactly this perturbation plays out, under the assumption that firm productivity is uniform. In the quantitative section, we find similar results under more general distributional assumptions.

The rising minimum wage naturally speaks to Fact 1—the bottom-driven decline—and Fact 2—heterogeneous inequality trends across regions as a function of their initial effective bindingness of the minimum wage. Fact 3 of our empirical part establishes that the main driver behind Brazil’s inequality decline was a fall in the pass-through from firm productivity to pay as well as lower returns to measures of worker ability. The following proposition shows that our model also rationalizes this fact:

Proposition 1. *Suppose $p \sim U(p_0, \bar{p})$, then an increase in the minimum wage $w^{\min}$:*

1. *increases worker pay in markets for which the minimum wage binds relative to higher worker types:*

$$\partial \left[\frac{w(p, \theta; w^{\min})}{w(p, \theta'; w^{\min})} \right] / \partial w^{\min} > 0, \quad \forall \theta, \theta' : \theta < \frac{w^{\min}}{p_0} \leq \theta'$$

2. *reduces the productivity-pay gradient across firms in affected markets:*

$$\partial \left[\frac{\partial w(p, \theta; w^{\min})}{\partial p} \right] / \partial w^{\min} < 0, \quad \forall \theta : \theta < \frac{w^{\min}}{p_0}$$

3. *reduces the returns to ability across workers in affected markets:*

$$\partial \left[\frac{\partial w(p, \theta; w^{\min})}{\partial \theta} \right] / \partial w^{\min} < 0, \quad \forall \theta : \theta < \frac{w^{\min}}{p_0}$$

Proof. See Appendix B. □

The intuition for the first part of Proposition 1 is straight-forward: An increase in the minimum wage reduces the monopsony power of firms in markets where it binds, which raises average pay in those markets relative to markets where the minimum wage does not bind. Since the minimum wage is more likely to bind in low ability markets, this reduces the pay gap between high and low ability workers.

To understand the second part, note that in binding markets a rise in the minimum wage forces the least productive firms to increase pay one-for-one (or exit). Firms with higher productivity profitably retain a positive wage premium by adjusting wages upwards, but by a smaller amount than firms below them, leading the minimum wage effects to fade out towards higher productivity levels. As a result, the pay-productivity gradient across firms falls.

Similarly the third result shows that a rise in the minimum wage increases wages by the greatest amount among the lowest worker types, for which the minimum wage is originally the most binding, thereby reducing the ability-pay gradient across workers.

6 Estimating the model

The previous section showed that the minimum wage can qualitatively rationalize our three facts on Brazil’s earnings inequality decline from 1996–2012. To quantitatively evaluate the effects of the rise in the minimum wage, we estimate our model to key moments in the microdata on Brazilian labor markets for the “pre-period” 1996–2000. To this end, we target cross-sectional moments of the joint distribution of worker and firm heterogeneity on the one hand, and information on worker type-specific transitions between employment states on the other hand.

6.1 Estimation strategy

Parameters guiding the job ladder structure of our model are identified off ordinal information on worker flows across firm pay ranks. Following Cahuc et al. (2006), we thus adopt a two-stage estimation procedure. In a first stage, we non-parametrically identify labor market mobility parameters guiding labor market transition rates. In a second stage we then employ the method of simulated moments via indirect inference (Smith, 1993) in order to estimate parameters guiding the distributions of worker ability and firm productivity.

First stage. In the first stage of our estimation routine, we use panel information on worker flows between employment states together with non-parametric estimates of the worker type-specific distribution of firm pay ranks in order to estimate the four labor market parameters of our model for each worker type θ : the separation rate, δ_θ , the job offer arrival rate from non-employment, λ_θ^u , the job offer arrival rate from employment, λ_θ^e , and the reservation wage, ϕ_θ .

To start with, classify workers into deciles according to their rank in the estimated AKM worker fixed effects distribution. We justify this approach in Figure 23 in the Appendix by showing that the correlation between underlying worker ability and estimated AKM worker effect is very high in our simulated model, above 0.97. We then construct a monthly worker panel from the RAIS data in order to calculate the fraction of formal sector entrants, exiters, and job-to-job switchers by worker type in every year between 1996 and 2012.²⁹ Following the same workers across employment states, we classify job-to-job (J2J) transitions, employment to non-employment (EN) transitions, and non-employment to employment (NE) transitions for each worker type θ at monthly frequency. We use this panel to estimate the four labor market parameters of interest by worker deciles for the “pre-period” 1996–2000.

First, the worker ability-specific monthly separation rate, δ_θ , is straight-forward to compute as the average rate of leaving the formal labor force:

$$\delta_\theta = \mathbb{E}(\text{exit} | \text{ability } \theta) \quad (11)$$

Second, to estimate the monthly rate of finding a job from non-employment λ_θ^u , we use a proportional hazards model in order to predict re-entry of workers that left formal sector employment within the previous 24 months. We invert the model in order to recover the employment finding rate using the relationship

$$\log \mathbb{P}(\# \text{ months until re-entry} \geq t | \text{ability } \theta) = t \times \log \pi_\theta^u \quad (12)$$

where $\pi_\theta^u = 1 - \lambda_\theta^u$ is the probability of remaining unemployed. We then solve for $\lambda_\theta^u = 1 - \exp\left(\widehat{\log \pi_\theta^u}\right)$, where $\widehat{\log \pi_\theta^u}$ denotes the ordinary least squares (OLS) estimate of the coefficient on t from equation (12).

²⁹Since we are unable to distinguish flows between formal sector employment on the one hand and unemployment, informal employment, or out of the labor force status on the other hand, we use the term “non-employment” to stand in for any of the latter terms.

Third, the job offer arrival rate on the job, λ_θ^e , cannot be directly inferred from observed job-to-job flows, since an employed worker only accepts offers paying more than their current employer. Our model, however, suggests that this parameter guides the speed of upwards employer mobility Jolivet et al. (2006), where we interpret the ranks of estimated AKM firm effects as the rungs of the job ladder, which different worker types agree on. Specifically, λ_θ^e is tightly linked to the distance between the cross-sectional wage distribution G and the wage offer distribution F :

$$G_\theta(w) = \frac{F_\theta(w)}{1 + \kappa_\theta^e (1 - F_\theta(w))} \quad (13)$$

where $\kappa_\theta^e = \lambda_\theta^e / \delta_\theta$ is the speed of workers of ability θ climbing the job ladder, defined as the probability of receiving an outside offer relative to the exogenous separation hazard. Thus we recover κ^e from equation (14) using non-parametric density function estimates³⁰ $\hat{G}(fe_\theta)$ of the cross-sectional firm effects distribution, as well as the firm effects offer distribution, $\hat{F}(fe_\theta)$, where the latter is approximated as the distribution of workers across firm effect ranks for new formal sector entrants. The nonparametric estimate of κ^e is then³¹

$$\hat{\kappa}^e = \frac{\hat{F}(fe_\theta) - \hat{G}(fe_\theta)}{(1 - \hat{F}(fe_\theta)) \hat{G}(fe_\theta)} \quad (14)$$

Using our earlier estimate of δ_θ , we use equation (14) to back out the implied value for λ_θ^e and hence the relative on-the-job search intensity $s_\theta = \lambda_\theta^e / \lambda_\theta^u$.

Fourth, we recall that $\max\{\phi_\theta, w^{min}\}$ is the lowest accepted wage by a worker of ability θ . This means that for the (relevant) range $\phi_\theta > w^{min}$ we can directly infer workers reservation wage as

$$\phi_\theta = \min_{i \in \theta} \{w_i\} \quad (15)$$

Note that we need not identify the reservation wage for workers with $\phi_\theta \leq w^{min}$ as for them the minimum wage will be the relevant binding constraint. In practice, to limit the role of measurement error, we implement equation (15) by identifying the first percentile of the earnings distribution conditional on being at or above the minimum wage and setting the reservation wage equal to that value.

³⁰We use a 1000 quantile bin approximation to the empirical cumulative distribution functions, although our results are unchanged for any reasonable bin number, and alternative estimation methods such as kernel density estimates.

³¹In Appendix ??, we discuss an alternative model-consistent way of estimating κ_θ^e , which produces similar estimates.

Finally, we interpolate the four labor market parameters that we estimated across worker ability deciles using a linear function for our model simulations using an arbitrarily high number of worker types θ on the computer.

Second stage. In the second stage of our estimation routine, we assume that worker ability is log normally distributed with mean μ and standard deviation σ , and that firm productivity Pareto distributed with tail parameter ζ (we normalize the scale parameter to one):

$$\log \theta \sim \mathcal{N}(\mu, \sigma^2), \quad \text{and} \quad p \sim \text{Pareto}(\zeta)$$

We show below that these distributional assumptions allow the model to well capture the empirical shape of the wage distribution.

Furthermore, we assume that the cost of creating jobs is on the following form,

$$c_\theta(v) = \frac{c_\theta v^{c_1+1}}{c_1 + 1}, \quad c_\theta, c_1 > 0$$

Without data on vacancies, we cannot separately identify match efficiency, χ , from the cost of creating jobs, c_θ . We hence normalize $\chi \equiv 1$. Furthermore, we cannot reliably estimate the curvature of the matching function, α , and hence we follow the literature to set $\alpha = 0.5$. Finally, the lack of data on vacancies prohibits us from estimating the curvature of the vacancy cost function, and hence we proceed under the quadratic cost case, $c_1 = 1$.

We estimate the remaining parameters of the model by indirect inference: average worker ability, μ , the dispersion in worker ability, σ , the shape of the firm productivity distribution, ζ , and the cost of creating jobs, c_θ . An appealing aspect of our model is its sparse parameterization. Heuristically, the following moments identify the following parameters: The bindingness of the minimum wage as measured by the minimum to mean log wage identifies average worker ability. The dispersion in AKM worker fixed effects identifies the dispersion in underlying worker ability, and the dispersion in AKM firm effects identifies the shape of the productivity distribution. Finally, the cost of creating jobs is identified by λ_θ^u that we estimated in the first step. Although each of these moments is particularly informative about one particular parameter, all parameters are jointly determined.

Our choice of target moments is motivated by our discussion in the previous section. We

argued there that absent a minimum wage and with identical labor market transition hazards across markets, wages in our model are log additively separable into log worker ability and a firm effect that is independent of worker ability. We showed that the latter is a strictly increasing transformation of firm productivity. Although an exact decomposition no longer prevails in our more general environment, it seems plausible that the dispersion in AKM firm and worker effects provides valuable information for identifying the underlying dispersion in worker ability and firm productivity. We show in Appendix D that these two moments indeed do appear to well identify these two parameters, in the sense that the distance between the relevant data and model moment quickly increases as we change the parameter of interest. The same is true with respect to average worker ability: the minimum-to-mean log wage in the model quickly departs from the data target as we change average worker ability. We discuss in greater detail our estimation routine in Appendix D.

6.2 Parameter estimates

Table 8 contains the resulting parameterization of worker type-specific labor market parameters that we use in order to simulate our model with many more worker types: the monthly separation rate δ_θ , the job finding rate from nonemployment λ_θ^u , the relative on-the-job search intensity s_θ , and the reservation wage ϕ_θ .³² We notice that while our estimate of the employment-to-nonemployment (EN) hazard from formal sector employment is similar to what is commonly found in the U.S., the estimated nonemployment-to-employment (NE) hazard is more in line with continental European countries. We find substantial heterogeneity in labor market transition rates by worker ability, particularly for the exit rate from formal sector employment. The EN hazard of the lowest decile of worker effects is over four times as high as for the highest decile of workers, while the NE hazard is 32 percent lower and relative search intensity while employed 53 percent lower. As we show below, this large heterogeneity in transition rates implies substantial sorting of more able workers to higher paying firms. Finally, the estimated reservation wage is increasing in worker ability.

³²Figure ?? in the Appendix shows estimated labor market parameters by worker ability decile for the period 1996-2000.

Table 8. Monthly labor market parameters

Description	Parameter	Value	Target
EN rate	Intercept	δ_0	0.039
	Slope	δ_1	-0.030
NE hazard	Intercept	λ_0^u	0.050
	Slope	λ_1^u	0.023
Relative search intensity	Intercept	s_0	0.192
	Slope	s_1	0.213
Reservation wage	Intercept	ϕ_0	-0.90
	Slope	ϕ_1	1.60

Notes: Targets are computed by AKM worker fixed effect decile for period 1996-2000. Model parameters over the normalized worker type interval $\theta \in [0, 1]$ are then filled in as a linear approximation to the estimated relationship.

Figure 7 plots the estimated cost of creating jobs that is required to match the differences in labor market hazard rates and the value of leisure across worker types. The relative cost of hiring a high ability worker (solid blue) is estimated to decline in worker ability, which is dictated by the higher job finding rate of high ability workers. The absolute cost (dashed red) is increasing, which is required by the model to avoid predicting a too large increase in the job finding rate with ability.

Figure 7. Estimated cost of creating jobs by worker type

Table 9 shows the estimated structural parameters of our model.

Table 9. Structural estimates of worker ability and firm productivity distributions

Description	Parameter	Value	Target
Mean worker ability	μ	1.81	Min-to-mean earnings ratio
Variance of worker ability	σ	0.50	Variance of AKM worker effect
Tail index of firm productivity	ζ	6.50	Variance of AKM firm effect

Note: Mean and variance of worker ability refer to log-normal distribution parameters. Tail index of firm productivity refers to shape parameter of the Pareto distribution, with mean firm productivity normalized to one. See text for details.

6.3 Model fit

Our estimated model successfully replicates both cross-sectional and longitudinal facts relating to the distribution of earnings and labor market dynamics in Brazil during the “pre-period” 1996–2000.

Figure 8 plots the empirical earnings distribution in panel (a) and the model-generated earnings distribution in panel (b). Since we do not target the residual in the AKM decomposition, the overall variance of earnings is somewhat under-predicted. Yet the shape of the distribution is well matched by our model, despite only targeting three moments of the underlying distributions of worker ability and firm productivity.

Figure 8. Earnings distribution in the model and data, 1996-2000

Table 10 compares the model fit in terms of estimated AKM components. By ways of our indirect inference step, the model replicates the variances of worker effects and firm effects exactly. The model also replicate a substantial share of the covariance between the worker effect and the

firm effect. The latter is not mechanical, but a result of the independently estimated (not targeted) labor market parameter heterogeneity across worker types, with higher ability workers traveling up the job ladder more quickly. As noted above, the model does not capture the full raw dispersion in earnings in the data since we do not target the residual in the AKM regression.³³ Overall, the model replicates 98.5 percent of the empirical variance of log earnings, net of the residual term.

Table 10. Model fit through the lens of AKM variance decomposition, 1996-2000

	(1)	(2)
Component of AKM earnings decomposition	Data	Model
Total variance of log earnings	0.72	0.65
Variance of worker fixed effects	0.36	0.36
Variance of firm fixed effects	0.17	0.16
2×Covariance b/w workers and firms	0.14	0.12
Residual variance	0.06	0.00
Worker years	90.2	0.43
R^2	0.92	100.0

Note: Variance decomposition is based on AKM regression $\log y_{ijt} = \alpha_i^p + \alpha_{j(i,t)}^p + \gamma_t + \varepsilon_{it}$. Variance of time effects and covariance terms involving time effects are small and omitted for brevity. See text for details.

The model also replicates two key exercises that the empirical literature tends to conduct to verify the AKM methodology, which we view as a validation of the model given that they are not targeted. First, as documented by [Alvarez et al. \(2016\)](#), wage gains and losses of workers switching up and down the firm ladder are roughly symmetric in Brazil. The left pane of Figure 9 reproduces their results for the 1996–2000 sub-period by plotting wages of workers that switch out of the first and fourth quartile of firm effects up to three years prior to the switch and two years after.³⁴ The right pane of Figure 9 plots the model-generated data. The model captures both the qualitative and quantitative aspects of the data well. Wages increase for workers that switch up the ladder and decline for those that move down. Furthermore, the model matches well the fact that average wages of those who move up to quartile j are lower than average wages of those who move down to quartile j . This is surprising in light of the fact that a worker can only move down the ladder by starting over from the bottom. As a result, we might for instance have expected workers who move from the fourth to the third quartile to have the same wage on average as

³³It is easy to add white noise to earnings in order to fit also the residual, but since this has no major impact on our results we refrain from doing so.

³⁴Following the methodology of [Card et al. \(2013\)](#), we produce this graph both in the model and data conditioning on the worker having been employed by the same employer for the past three years prior to the switch and the staying with the same employer for the two years after the switch. To avoid clogging up the graph, we only include switchers out of the first and fourth quartiles, but the other quartiles follow a similar pattern.

workers who move into the third quartile from the first quartile. The reason this is not true in the model is sorting. The average worker who makes a switch from the fourth to the third quartile is of higher ability than the average worker who switches from the first to the third quartile, and as a result the former earns a higher wage than the latter conditional on moving into the same quartile.

Figure 9. Wage gains from switching employer for period 1996–2000, data (left) and model (right)

Second, the empirical literature tends to investigate the behavior of the average residual from the AKM regression over worker and firm effects. The left pane of Figure 10 reproduces results from [Alvarez et al. \(2016\)](#) by plotting the average AKM residual by deciles of worker and firm effects in the 1996–2000 subperiod. The right pane shows the same figure on model-generated data. We note three things. First, the model captures well the key feature of the data that over most of the support, the average residual is close to zero. Second, the model qualitatively matches the fact that the lowest worker effect deciles have a positive residual while employed at the lowest paying firms. In the model, this is driven by the minimum wage pushing up wages of the lowest paid workers when they are employed at the lowest paying firms. Finally, the model substantially overstates this pattern at the bottom of the distribution. This pattern becomes stronger in both the model and the data as the minimum wage is raised.

6.4 Policy experiment in the model

To evaluate the impact of a rise in the minimum wage, we consider the following experiment: we hold all parameters fixed at their initial estimated values, and change only the minimum wage in

Figure 10. AKM residual plot for period 1996–2000

line with the data.³⁵ We calibrate the increase in the minimum wage to match the growth in the productivity adjusted real minimum wage between 1996–2000 and 2008–2012, which we compute in the following way. The average real minimum wage (in 2012 values) is 384 Reais in 1996–2000 and 701 in 2008–2012, implying an 60.2 log point growth in real minimum wages. Average log value added per worker grows by a total of 15.4 log points between 1996–2000 and 2008–2012. Thus, we estimate that the real, productivity adjusted minimum wage grew by 44.7 log points between 1996–2000 and 2008–2012.³⁶ This implies a hike in the minimum wage from 0.189 to 0.315 or roughly 67 percent. We evaluate the implications for income inequality of imposing this higher minimum wage through the lens of our model.

³⁵The assumption that the reservation wage remains constant in response to a change in the minimum wage deserves special mentioning, given that it is an equilibrium outcome rather than a parameter. This assumption is innocuous for the following reasons. Suppose first that the minimum wage when raised becomes binding in market θ . In this case, the reservation wage is no longer relevant, and it is without loss of generality to disregard any changes to it. Second, suppose that the minimum wage remains non-binding in market θ . In this case, nothing has changed in market θ , leaving the reservation wage unchanged.

³⁶For robustness, we also explored alternative targets for the increase in the minimum wage, including the growth rate of the minimum wage relative to productivity growth in Brazil’s services, commerce, and construction sectors (for which we have firm-level productivity data); or relative to growth in aggregate output per capita from national accounts. These alternative targets imply similar increases in the minimum wage and therefore lead to comparable results.

7 Quantitative results

In order to isolate the effect of the rise in the minimum wage on the earnings distribution, we consider the following counterfactual experiment. We hold all parameters fixed at their initial estimates for the period 1996–2000 and change only the minimum wage in the model to match the growth in the productivity adjusted minimum wage between 1996–2000 and 2008–2012 in Brazil over this period.

We first present results on the aggregate impact of a change in the minimum wage on earnings inequality, addressing our first empirical fact that the decline in inequality was most pronounced at the bottom of the distribution, yet widespread. Second, we show that the model correctly captures our second empirical fact about the co-variation between bottom tail inequality and the bindingness of the minimum wage across Brazilian states over time. Third, we decompose the decline in inequality in the model in the same way as in the data into a worker and a firm component, and we study changes in the pass-through from productivity to the firm component of pay and from worker ability to the worker component. We find that our model captures well our third empirical fact that the decline was driven by a fall in the pass-through from fundamentals to pay. Finally, we discuss the model’s predictions for changes in job finding rates and unemployment.

7.1 Effect throughout the earnings distribution

Table 11 compares log percentile ratios in the model with the data in the 1996–2000 and 2008–2012 subperiods. The model does a very good job at matching the empirical compression in earnings at different percentiles in response to an increase in the minimum wage. The model predicts roughly 70 percent of the compression in inequality at the bottom of the distribution, and about 50 percent at the top of the distribution. For instance, the 50–10 log ratio compresses by 31 log points in the data versus 21 log points in the model (or 68 percent) whereas the log 90–50 ratio compresses by six log points in the model versus 13 log points in the data (or 46 percent). The model displays a declining amount of compression the further up the wage distribution we go, successfully replicating Fact 1 from our empirical section.

Table 11. Compression in log percentile ratios of earnings distribution

	1996–2000		2008–2012		Change		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Data	Model	Data	Model	Data	Model	% Explained
P50-P10	0.86	0.95	0.55	0.74	-0.31	-0.21	68%
P50-P25	0.48	0.59	0.33	0.48	-0.15	-0.11	73%
P75-P50	0.60	0.67	0.50	0.62	-0.10	-0.05	50%
P90-P50	1.30	1.21	1.17	1.15	-0.13	-0.06	46%

7.2 Spill-over effects identified in cross-state variation

In order to investigate the model’s ability to reproduce our second empirical fact that bottom tail inequality is lower in states with a higher bindingness of the minimum wage, we implement the following minor change to our above methodology. Instead of estimating the model to fit the aggregate economy in 1996–2000, we estimate it to fit the least binding state in that subperiod. The model again fits the data well initially. We subsequently re-estimate average worker ability in each state to hit the initial level of the bindingness of the minimum in that state, while holding all other parameters fixed at their estimated values for the least binding state. Finally, we change the minimum wage as in the data over time, while holding all parameters fixed at their initial values. This approach aligns closely with the spirit of [Lee \(1999\)](#) that there is an underlying latent distribution of wages that is shared across all states, but that this is differentially masked by a federal minimum wage due to differences in wage levels across states.

Figure 11 plots the result of this exercise. The top left pane plots the relationship between bottom tail inequality, measured by the log 50-10 ratio, and the bindingness of the minimum wage, measured by the log median to minimum ratio, across Brazilian states over time. The right pane repeats this exercise on model generated data. There is clear downward sloping relationship that grows more pronounced as the minimum wage is raised, suggesting that the minimum has an impact on bottom tail inequality. Furthermore, the model captures this relationship well, both qualitatively and quantitatively. For comparison, the bottom two panes show that in both the model and data there is only a weak relationship between the bindingness of the minimum wage and top inequality, measured by the log 90-50 ratio.

Figure 11. Inequality and bindingness of minimum wage across Brazilian states over time

7.3 AKM decomposition and the sources of the decline

Figure 12 plots the variance of worker effects, firm effects and their covariance in the model and the data in each of the four subperiods. The model reproduces the entirety of the fall in the variance of the worker effect and the covariance, and matches close to half of the fall in the variance of the firm component. As in the data, the correlation between worker and firm effects remains stable. Given significant evidence from other countries that technological change over the last two decades has increased the return to ability (so called *skill-biased technical change*), we find it plausible that other forces served to increase the dispersion in person effects over this period in the data.

Figure 12. AKM decomposition in model and data

Table 12 presents numbers on the 1996–2000 and 2008–2012 subperiods.

Table 12. AKM decomposition in data and model

	1996–2000		2008–2012		Change		
	(1) Data	(2) Model	(3) Data	(4) Model	(5) Data	(6) Model	(7) % Explained
Total variance of log earnings	0.72	0.65	0.52	0.51	-0.20	-0.14	70%
Variance of worker fixed effects	0.36	0.37	0.31	0.30	-0.05	-0.07	140%
Variance of firm fixed effects	0.17	0.16	0.08	0.12	-0.09	-0.04	44%
2×Covariance b/w workers and firms	0.14	0.12	0.10	0.09	-0.04	-0.03	75%
Residual variance	0.06	0.00	0.04	0.00	-0.02	0.00	0%

In the model, the underlying distribution of worker ability and firm productivity is held constant by construction. The former, however, may change in response to some workers experiencing relative falls in their employment rates (in the extreme case in the form of permanent exit from the formal sector labor market). The latter may change in response to relative changes in the amount of vacancies posted by different firms, again in the extreme case in the form of permanent exit from some markets by some firms. To investigate to what extent the fall in inequality is driven by changes in the distribution of active workers and firms relative to changes in the return to worker ability and firm productivity, we consider two counterfactual scenarios. In the first, we hold constant the variance of worker and firm productivity and allow only the pass-through to change, while in the other we hold the pass-through constant and allow only the variance of ability and productivity to change.

The left pane of Figure 13 plots the percentage fall in the explainable component of the AKM

firm effect that is driven by a changing pass-through from productivity to pay in the data (solid blue) and model (dashed red). In the data, the component of pay that is accounted for by value added falls by over 60 percent over this period, while in the model the decline is a more moderate 25 percent. In absolute numbers, the variance of the explainable part falls by 0.05 log points in the data and 0.03 log points in the model. In neither the model nor the data is there much compression due to less dispersion in underlying fundamentals.

The right pane of Figure 13 plots the percentage fall in the explainable component of the AKM worker effect that is driven by a changing pass-through from worker ability to pay in the data (solid blue) and model (dashed red). Since the model does not contain as easily interpretable measures of ability as education and age, we construct this by regressing the worker component of pay on a linear in log ability. The explainable component falls by 25 percent in the data and 20 percent in the model. Also on the worker side is there no meaningful compression due to changes in the underlying distribution in worker ability in the model or the data.

Figure 13. Fall in inequality due to declining pass-through in data (solid blue) and model (dashed red), firms (left) and workers (right)

We conclude from these exercises that the model is able to successfully reproduce roughly half of the fall in the dispersion of the firm component of pay, and that in line with the data this is driven by a lower pass-through from firm productivity to pay. It somewhat overpredicts the compression in the worker effects, but again correctly captures the fact that this is driven by a lower return to worker ability, not changes in the underlying distribution of worker ability.

7.4 Employment effects

The increase in the minimum wage reduces firms' profit margins and hence vacancy creation. As a result, the job finding rate of workers falls. As we showed in the previous section, however, the model predicts only small effects on the composition of the workforce, suggesting that such general equilibrium effects are relatively minor. This is verified by Figure 14, which plots the job finding rates, λ_{θ}^u , in each period as a function of worker rank in the worker effect distribution (normalized to be on the unit interval). The left pane plots the predicted finding rates in the model, while the right pane plots those in the data.

The job finding rates in the model fall in response to the hike in the minimum wage, with the most pronounced effect at the bottom of the worker ability distribution. The fall, however, is modest throughout the distribution, with the bottom decile of workers experience a 10 percent decline in their job finding rate. The data, on the other hand, display a general increase in the job finding rates over this period in Brazil, potentially as a result of favorable economic shocks over this period. We note, though, that in line with the predictions of the model, the data display a relative steepening of the job finding rates with worker ability. In relative terms, the job finding rate of the bottom of the distribution fell by 11 percent relative to the top of the distribution.

Figure 14. Job finding rates by worker ability rank, by sub-period in model (left) and data (right)

8 Conclusion

In this paper, we analyzed sources of earnings inequality dynamics in general and the role of the minimum wage specifically. The starting point of our investigation were three key facts about Brazil, which experienced a rapid decline in earnings inequality between 1996 and 2012. Brazil's overall decline in earnings inequality was driven from the bottom. We find that one quarter of this decline stems from a weaker degree of pass-through from firm productivity to wages, and another quarter of the decline is attributable to falling pay differences due to unobserved worker characteristics.

To investigate the contribution of the minimum wage to these facts, we built a search model in the spirit of [Burdett and Mortensen \(1998\)](#), extended with heterogeneous firms and workers. The key feature of the model were spillover effects of the minimum wage due to monopsonistic competition among firms for workers. We characterize the equilibrium of this model and showed that the minimum accounts qualitatively for our documented facts.

Estimating the model on Brazilian microdata, we are also successful in explaining a large share of the overall inequality decline and quantitatively accounting for the three facts. Consistent with the observed compression of earnings, a large share of the inequality decline in our model is due to indirect effects of the minimum wage, resulting in a lower productivity-pay gradient across firms and lower returns to worker ability.

While the minimum wage may affect many other outcomes of interest ([Card and Krueger, 1994](#); [Manning, 2005](#); [Harasztosi and Lindner, 2015](#)), we have focused our analysis on the effects of the minimum wage on the earnings distribution. Although the key mechanism in our model is a general one and relies only on the inter-dependence between firms' wage offers, a key question is to what extent the Brazilian experience carries over to other economies such as the United States, where policy makers currently debate an increase in the minimum wage from 7.25 to 15.00 dollars. Our analysis sheds new light on one aspect of this question and suggests that the effects on earnings inequality will depend crucially on the structural parameters guiding the between-firm competition among firms for employees in those markets. Assessing the strength of this channel for other economies as well as for alternative policies including unemployment insurance, employment protection legislation, and non-discrimination laws would shed further light on the degree to which labor market dynamics can amplify the effects of policy on earnings inequality.

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Appendix

A Empirics

A.1 Inequality trends in Brazilian and U.S. household survey data

To put the magnitude of Brazil's inequality decline into context, Figure 15 plots the evolution of a common inequality measure, the variance of log earnings, from 1996–2012. Data for Brazil come from the largest national household survey, the *Pesquisa Nacional por Amostra de Domicílios* (PNAD). Data for the U.S. are based on the *March Current Population Survey* (CPS). In both datasets, earnings inequality is computed over log earnings for male and female labor market participants of age 18–64. The income concept is taken to be labor earnings in the week preceding the survey, and the top and bottom 1% of all observations are dropped to control for outliers.

Figure 15 shows that while the variance of log earnings in the Brazilian household survey dropped by 27 log points from 1996 to 2012, it rose by six log points in the U.S. household data over the same period. Thus, Brazil's inequality decline is of a relatively large magnitude, both within the Brazilian context and in the comparison with the U.S. experience.

Figure 15. Evolution of variance of log earnings in Brazil and the U.S., 1996–2012

A.2 Additional facts about Brazil's inequality decline

Fact 4. *The inequality decline featured compression up to the 90th percentile of the earnings distribution. Yet all parts of the distribution experienced earnings growth between 1996 and 2012.*

Figure 16 plots the evolution of normalized (to zero in 1996) log percentile ratios, all relative to the median of the earnings distribution. There was pronounced catch-up throughout most of the earnings distribution, but more rapidly between the median and the bottom percentiles, as seen by the drop of the bottom percentile ratios relative those at the top. In fact, we see that above the 90th percentile there was little or no compression, evidenced by the log percentile ratio lines coinciding in the graph.

Figure 16. Normalized evolution of earnings percentile ratios, 1996–2012

Figure 17. Inequality evolution within states across years, 1996–2012

Figure 18. Inequality evolution within industries across years, 1996–2012

Figure 19. Inequality evolution within education groups across years, 1996–2012

The decline in the productivity-pay gradient across firms is nicely illustrated in Figure 20, which shows the relative firm effects in pay for decile bins of the worker-weighted value added distribution across firms in the PIA data.

Figure 20. Lower firm productivity-pay gradient between initial and final periods

A.3 Evolution of the real minimum wage in Brazil

Figure 21. Evolution of the real minimum wage in Brazil, 3-month running averages (Source: IPEA)

Figure 22. Empirical earnings distributions by year, 1996-2012

B Proofs

B.1 Proof of Proposition 1

Assume that $p \sim U(\underline{p}, \bar{p})$. Then we can write the piece rate $\tilde{w}$ offered by a firm with productivity p in market θ as

$$\begin{aligned} \frac{\tilde{w}(p, \theta; w^{min})}{\theta} &= p - \int_{\underline{p}(\theta; w^{min})}^p \left[\frac{1 + \kappa^e (1 - F_\theta(p))}{1 + \kappa^e (1 - F_\theta(x))} \right]^2 dx \\ &= p - \int_{\underline{p}(\theta; w^{min})}^p \left[\frac{1 + \kappa^e \left(\frac{\bar{p} - p}{\bar{p} - \underline{p}(\theta; w^{min})} \right)}{1 + \kappa^e \left(\frac{\bar{p} - x}{\bar{p} - \underline{p}(\theta; w^{min})} \right)} \right]^2 dx \end{aligned}$$

From here, we consider two cases.

Case 1. $\theta \leq \frac{w^{min}}{\bar{b}}$ In this first case, for markets affected by the minimum wage, we can write:

$$\begin{aligned} \frac{\tilde{w}(p, \theta; w^{min})}{\theta} &= p - \int_{w^{min}/\theta}^p \left[\frac{1 + \kappa^e \left(\frac{\bar{p} - p}{\bar{p} - w^{min}/\theta} \right)}{1 + \kappa^e \left(\frac{\bar{p} - x}{\bar{p} - w^{min}/\theta} \right)} \right]^2 dx \\ &= p - \left[1 + \kappa^e \left(\frac{\bar{p} - p}{\bar{p} - w^{min}/\theta} \right) \right]^2 \int_{w^{min}/\theta}^p \frac{dx}{\left[1 + \left(\frac{\kappa^e}{\bar{p} - w^{min}/\theta} \right) \bar{p} - \left(\frac{\kappa^e}{\bar{p} - w^{min}/\theta} \right) x \right]^2} \\ &= p - \left[1 + \kappa^e \left(\frac{\bar{p} - p}{\bar{p} - w^{min}/\theta} \right) \right]^2 \left\{ \frac{1}{\left(\frac{\kappa^e}{\bar{p} - w^{min}/\theta} \right) \left[1 + \left(\frac{\kappa^e}{\bar{p} - w^{min}/\theta} \right) (\bar{p} - x) \right]} \right\}_{x=w^{min}/\theta}^p \\ &= p - \left[1 + \kappa^e \left(\frac{\bar{p} - p}{\bar{p} - w^{min}/\theta} \right) \right]^2 \\ &\quad \times \left\{ \frac{1}{\left(\frac{\kappa^e}{\bar{p} - w^{min}/\theta} \right) \left[1 + \left(\frac{\kappa^e}{\bar{p} - w^{min}/\theta} \right) (\bar{p} - p) \right]} - \frac{1}{\left(\frac{\kappa^e}{\bar{p} - w^{min}/\theta} \right) \left[1 + \left(\frac{\kappa^e}{\bar{p} - w^{min}/\theta} \right) (\bar{p} - w^{min}/\theta) \right]} \right\} \\ &= p - \left\{ \frac{1 + \kappa^e \left(\frac{\bar{p} - p}{\bar{p} - w^{min}/\theta} \right)}{\frac{\kappa^e}{\bar{p} - w^{min}/\theta}} - \frac{\left[1 + \kappa^e \left(\frac{\bar{p} - p}{\bar{p} - w^{min}/\theta} \right) \right]^2}{\frac{\kappa^e}{\bar{p} - w^{min}/\theta} (1 + \kappa^e)} \right\} \\ &= p - \left(\frac{\bar{p} - \frac{w^{min}}{\theta} + \kappa^e (\bar{p} - p)}{1 + \kappa^e} \right) \left(\frac{p - \frac{w^{min}}{\theta}}{\bar{p} - \frac{w^{min}}{\theta}} \right) \end{aligned}$$

Case 2. $\theta \leq \frac{w^{min}}{\bar{b}}$ In this second case, for markets affected by the minimum wage, we have:

$$\begin{aligned}
\frac{\bar{w}(p, \theta; w^{min})}{\theta} &= p - \int_{\bar{b}}^p \left[\frac{1 + \kappa^e \left(\frac{\bar{p}-p}{\bar{p}-\tilde{b}} \right)}{1 + \kappa^e \left(\frac{\bar{p}-x}{\bar{p}-\tilde{b}} \right)} \right]^2 dx \\
&= p - \left[1 + \kappa^e \left(\frac{\bar{p}-p}{\bar{p}-\tilde{b}} \right) \right]^2 \int_{\bar{b}}^p \frac{dx}{\left[1 + \left(\frac{\kappa^e}{\bar{p}-\tilde{b}} \right) \bar{p} - \left(\frac{\kappa^e}{\bar{p}-\tilde{b}} \right) x \right]^2} \\
&= p - \left[1 + \kappa^e \left(\frac{\bar{p}-p}{\bar{p}-\tilde{b}} \right) \right]^2 \left\{ \frac{1}{\left(\frac{\kappa^e}{\bar{p}-\tilde{b}} \right) \left[1 + \left(\frac{\kappa^e}{\bar{p}-\tilde{b}} \right) (\bar{p}-x) \right]} \right\}_{x=\bar{b}}^p \\
&= p - \left[1 + \kappa^e \left(\frac{\bar{p}-p}{\bar{p}-\tilde{b}} \right) \right]^2 \left\{ \frac{1}{\left(\frac{\kappa^e}{\bar{p}-\tilde{b}} \right) \left[1 + \left(\frac{\kappa^e}{\bar{p}-\tilde{b}} \right) (\bar{p}-p) \right]} - \frac{1}{\left(\frac{\kappa^e}{\bar{p}-\tilde{b}} \right) \left[1 + \left(\frac{\kappa^e}{\bar{p}-\tilde{b}} \right) (\bar{p}-\tilde{b}) \right]} \right\} \\
&= p - \left\{ \frac{1 + \kappa^e \left(\frac{\bar{p}-p}{\bar{p}-\tilde{b}} \right)}{\frac{\kappa^e}{\bar{p}-\tilde{b}}} - \frac{\left[1 + \kappa^e \left(\frac{\bar{p}-p}{\bar{p}-\tilde{b}} \right) \right]^2}{\frac{\kappa^e}{\bar{p}-\tilde{b}} (1 + \kappa^e)} \right\} \\
&= p - \left(\frac{\bar{p}-\tilde{b} + \kappa^e (\bar{p}-p)}{1 + \kappa^e} \right) \left(\frac{p-\tilde{b}}{\bar{p}-\tilde{b}} \right)
\end{aligned}$$

Thus, we can write the wages offered at any firm in the economy with a minimum wage as follows:

$$w(p, \theta; w^{min}) = \begin{cases} p\theta - \theta \left(\frac{\bar{p} - \frac{w^{min}}{\theta} + \kappa^e (\bar{p}-p)}{1 + \kappa^e} \right) \left(\frac{p - \frac{w^{min}}{\theta}}{\bar{p} - \frac{w^{min}}{\theta}} \right) & \text{for } \theta \leq \frac{w^{min}}{\bar{b}} \\ p\theta - \theta \left(\frac{\bar{p}-\tilde{b} + \kappa^e (\bar{p}-p)}{1 + \kappa^e} \right) \left(\frac{p-\tilde{b}}{\bar{p}-\tilde{b}} \right) & \text{otherwise} \end{cases}$$

Taking derivatives of this expression:

$$\begin{aligned}
\frac{\partial w(p, \theta; w^{min})}{\partial p} &= \begin{cases} \frac{2\theta\kappa^e \left(p - \frac{w^{min}}{\theta} \right)}{(1 + \kappa^e) \left(\bar{p} - \frac{w^{min}}{\theta} \right)} > 0 & \text{for } \theta \leq \frac{w^{min}}{\bar{b}} \\ \frac{2\theta\kappa^e (p-\tilde{b})}{(1 + \kappa^e) (\bar{p}-\tilde{b})} > 0 & \text{otherwise} \end{cases} \\
\partial \left[\frac{\partial w(p, \theta; w^{min})}{\partial p} \right] / \partial w^{min} &= \begin{cases} -\frac{2\kappa^e (\bar{p}-p)}{(1 + \kappa^e) \left(\bar{p} - \frac{w^{min}}{\theta} \right)^2} < 0 & \text{for } \theta \leq \frac{w^{min}}{\bar{b}} \\ 0 & \text{otherwise} \end{cases}
\end{aligned}$$

To prove the second part of the proposition, consider two worker types θ_i and θ_j with $\theta_i > \theta_j$ and a firm p active in both markets. Suppose a binding minimum wage is imposed and consider

the difference in the firm component of pay between firm p to the two types of workers

$$\begin{aligned}
& r(p, \theta_j; w^{min}) - r(p, \theta_i; w^{min}) \\
&= \int_{\frac{w^{min}}{\theta_j}}^p \left[\frac{1 - \Gamma\left(\frac{w^{min}}{\theta_i}\right) + \kappa^e (1 - \Gamma(p))}{1 - \Gamma\left(\frac{w^{min}}{\theta_i}\right) + \kappa^e (1 - \Gamma(x))} \right]^2 dx - \int_{\frac{w^{min}}{\theta_j}}^p \left[\frac{1 - \Gamma\left(\frac{w^{min}}{\theta_j}\right) + \kappa^e (1 - \Gamma(p))}{1 - \Gamma\left(\frac{w^{min}}{\theta_j}\right) + \kappa^e (1 - \Gamma(x))} \right]^2 dx \\
&> \int_{\frac{w^{min}}{\theta_j}}^p \left\{ \left[\frac{1 - \Gamma\left(\frac{w^{min}}{\theta_i}\right) + \kappa^e (1 - \Gamma(p))}{1 - \Gamma\left(\frac{w^{min}}{\theta_i}\right) + \kappa^e (1 - \Gamma(x))} \right]^2 - \left[\frac{1 - \Gamma\left(\frac{w^{min}}{\theta_j}\right) + \kappa^e (1 - \Gamma(p))}{1 - \Gamma\left(\frac{w^{min}}{\theta_j}\right) + \kappa^e (1 - \Gamma(x))} \right]^2 \right\} dx
\end{aligned}$$

It is hence sufficient to show that for $x \in \left[\frac{w^{min}}{\theta_j}, p\right]$:

$$\begin{aligned}
& \left[\frac{1 - \Gamma\left(\frac{w^{min}}{\theta_i}\right) + \kappa^e (1 - \Gamma(p))}{1 - \Gamma\left(\frac{w^{min}}{\theta_i}\right) + \kappa^e (1 - \Gamma(x))} \right]^2 \geq \left[\frac{1 - \Gamma\left(\frac{w^{min}}{\theta_j}\right) + \kappa^e (1 - \Gamma(p))}{1 - \Gamma\left(\frac{w^{min}}{\theta_j}\right) + \kappa^e (1 - \Gamma(x))} \right]^2 \\
& \iff \Gamma\left(\frac{w^{min}}{\theta_j}\right) [\Gamma(p) - \Gamma(x)] \geq \Gamma\left(\frac{w^{min}}{\theta_i}\right) [\Gamma(p) - \Gamma(x)]
\end{aligned}$$

For $x = p$ this inequality is clearly satisfied. For any $x < p$, since by assumption $\theta_i > \theta_j$ it follows that $\Gamma\left(\frac{w^{min}}{\theta_j}\right) \geq \Gamma\left(\frac{w^{min}}{\theta_i}\right)$ by virtue of Γ being a CDF.

To prove the final part of the proposition, note that

$$\mathbb{E}_{\theta_i}(p; w^{min}) = \mathbb{E}\left(p \mid p \geq \max\left\{\frac{w^{min}}{\theta_i}, p_0\right\}\right)$$

Clearly,

$$\theta_i > \theta_j \implies \mathbb{E}\left(p \mid p \geq \max\left\{\frac{w^{min}}{\theta_i}, p_0\right\}\right) \leq \mathbb{E}\left(p \mid p \geq \max\left\{\frac{w^{min}}{\theta_j}, p_0\right\}\right)$$

This concludes the proof of Propositions ?? and ??.

C Solution algorithm

This section discusses the solution algorithm we employ to solve and estimate the model. Under the assumption that the vacancy cost is quadratic, we can write the first-order conditions for the

firm as

$$c_0 v_\theta(p) = T_\theta (p - \tilde{w}(p)) \left(\frac{1}{\delta_\theta + s_\theta \lambda_\theta^u (1 - F_\theta(\tilde{w}(p)))} \right)^2$$

and

$$1 = (p - \tilde{w}_\theta(p)) \frac{2s_\theta \lambda_\theta^u f_\theta(\tilde{w}_\theta(p))}{\delta_\theta + s_\theta \lambda_\theta^u (1 - F_\theta(\tilde{w}_\theta(p)))}$$

where

$$T_\theta = \theta \frac{u_\theta \lambda_\theta^u (\delta_\theta + s_\theta \lambda_\theta^u)}{V_\theta}$$

Define $h_\theta(p) = F_\theta(w_\theta(p))$ so that $f_\theta(w_\theta(p)) = h'(p)/w'_\theta(p)$ and $v_\theta(p) = \frac{V_\theta h'(p)}{\gamma(p)}$. Substituting this into the first-order equations, we have

$$h'_\theta(p) = \frac{T_\theta}{c_0 V_\theta} (p - \tilde{w}(p)) \left(\frac{1}{\delta_\theta + s_\theta \lambda_\theta^u (1 - h_\theta(p))} \right)^2 \gamma(p) \quad (16)$$

and

$$w'_\theta(p) = \frac{2s_\theta \lambda_\theta^u T_\theta}{c_0 V_\theta} (p - \tilde{w}_\theta(p))^2 \left(\frac{1}{\delta_\theta + s_\theta \lambda_\theta^u (1 - h_\theta(p))} \right)^3 \gamma(p) \quad (17)$$

Our empirical section estimates λ_θ^u , δ_θ and s_θ . Under these parameter estimates, the unemployment rate is given by

$$u_\theta = \frac{\delta_\theta}{\delta_\theta + \lambda_\theta^u}$$

and total vacancies by

$$V_\theta = \lambda_\theta^{\frac{1}{\alpha}} (u_\theta + s_\theta (1 - u_\theta))$$

Having obtained unemployment and total vacancies, we solve the system of differential equations (16)–(17) for a given guess for the cost of creation jobs, c_0 , subject to the boundary conditions

$$\tilde{w}_\theta(\underline{p}_\theta) = \frac{\max\{\phi_\theta, w^{min}\}}{\theta}, \quad \text{and} \quad h(\underline{p}) = 0$$

We update the guess for the cost c_0 until the vacancy policy integrated across firms replicates the total amount of vacancies necessary to match empirical transition hazards.

We adopt the above algorithm when using the model in order to evaluate effects of a rise in the minimum wage. Instead of taking λ_θ^u from the data and iterating over c_0 to match that, we keep c_0 fixed at its “pre-period” estimate and then loop over the new job finding rate λ_θ^u until implied total vacancies are consistent with optimizing firm behavior in the post-period.

D Estimation

D.1 First stage

Figure 23. Relation between model heterogeneity and AKM estimates, 1996–2000

D.2 Second stage

As outlined in Section 6, we estimate the average worker ability, the dispersion in worker ability, the shape of the productivity distribution, and the cost of creating jobs by indirect inference. We implement this by repeatedly solving the model over a pre-specified grid for the first three parameters and recording the model-predicted values for the targeted moments. Within each loop, we iteratively solve for the cost of creating jobs that allows the model to match the UE hazard rate estimated in the first step of our estimation.

We use a 25 by 25 by 25 point grid in the three parameters of interest, defined over a sufficiently large domain. Solving and parallel-simulating the model 15,625 times is relatively efficient and runs for about 16 hours on a modern quad-core desktop computer. Our objective is to minimize the sum of squared log ratios between the log min-to-mean ratio, the variance of worker fixed effects, and the variance of firm fixed effects in the model and the data:

$$\left(\hat{\sigma}_\theta, \hat{\sigma}_p, w^{min}\right) = \operatorname{argmin}_{\sigma_\theta, \sigma_p, w^{min}} \left\{ W_{\alpha_i} \left[\frac{\operatorname{Var}(\alpha_i^D) - \operatorname{Var}(\alpha_i^M)}{\operatorname{Var}(\alpha_i^D)} \right]^2 + W_{\alpha_j} \left[\frac{\operatorname{Var}(\alpha_j^D) - \operatorname{Var}(\alpha_j^M)}{\operatorname{Var}(\alpha_j^D)} \right]^2 + W_{mM} \left[\frac{mM^D - mM^M}{mM^D} \right]^2 \right\}$$

with weights W_{α_i} , W_{α_j} , and W_{mM} , which we set equal to unity in our baseline estimations.³⁷

To verify that the grid is large enough as well as the uniqueness of the solution, we analyzed the behavior of the objective function in two dimensions at a time, fixing the third parameter at its estimated value. This is plotted in Figure 24. The variance of worker ability appears well identified, while there is some ambiguity in the worker ability-firm productivity shape dimension. Specifically, a higher mean worker ability—implying a less binding minimum wage—can to be compensated for by a higher shape of the firm productivity distribution—implying less disperse firm productivity.

We have re-evaluated the impact of an increase in the minimum wage for different combinations of (μ, ζ) that produce only a slightly worse fit with the data, and find very similar results.

Figure 24. Distance metric from estimation procedure for period 1996–2000

³⁷We have tried different weights without significantly changing our conclusions.